Quantified Gamow Shell Model interaction for *p*- (and maybe *sd*-) nuclei

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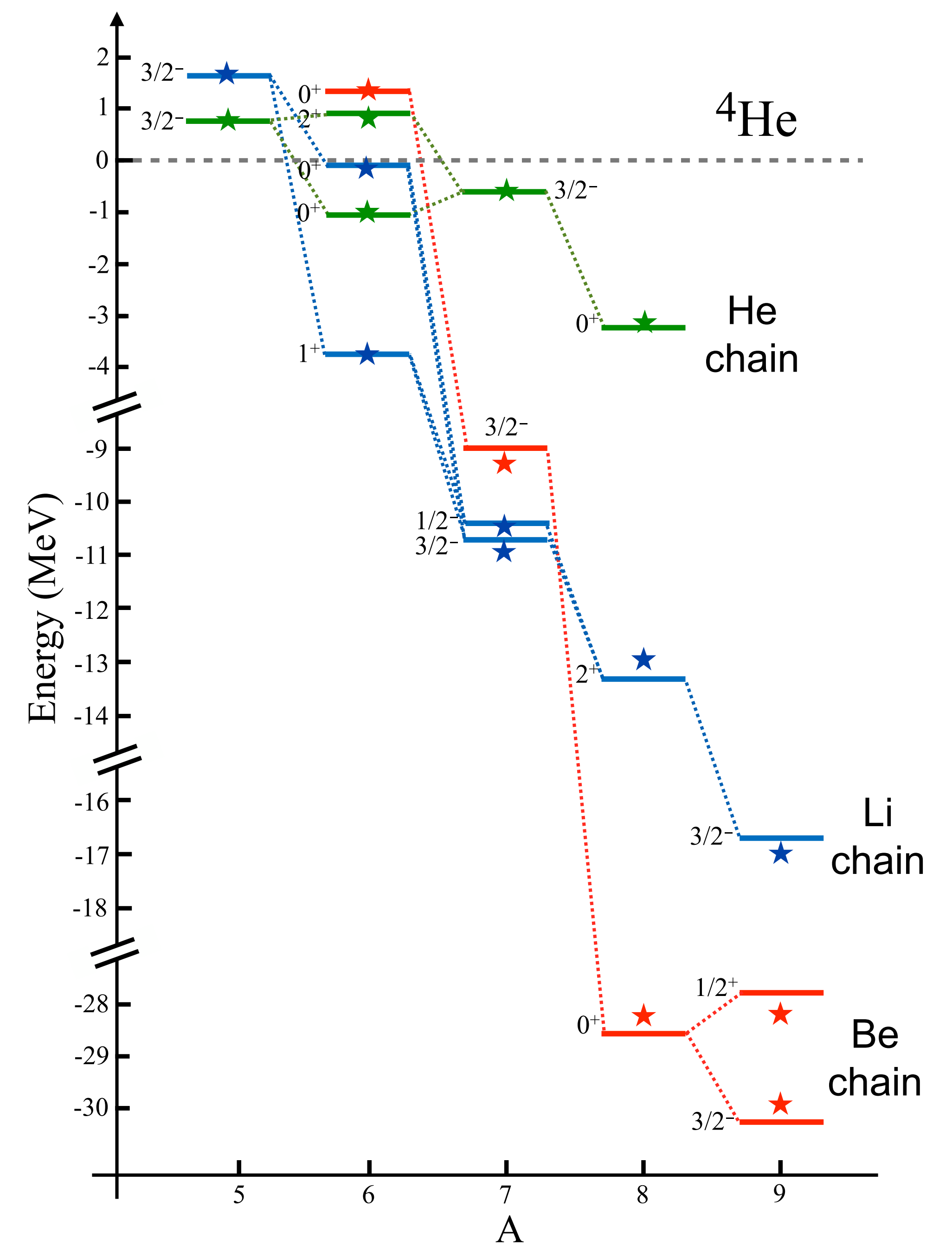
Phys. Rev. C 96, 054316 (2017)

We optimized an **effective interaction** within the **Gamow Shell Model (GSM)** framework, designed to describe a variety of **structure** (**bound + unbound**) and **reaction** observables across the **p-**nuclei (A ≃ 5 - 15)

Statistical studies were carried out to assess **statistical uncertainties and correlations.**

Now is the time for **applications** in collaboration with experimentalists:
Interplay theory ⇔ experiments to improve the interaction and make better predictions.



Outline

1. The framework: the Gamow Shell Model

2. The quantified GSM effective interaction

3. Applications
    - Correlation densities
    - Excited spectra

4. How to improve the interaction with different experimental data

5. Summary and outlook

The Framework: The Gamow Shell Model

N. Michel, W. Nazarewicz, M. Płoszajczak, T. Vertse, J. Phys. G: Nucl. Part. Phys. 36, 013101 (2009)

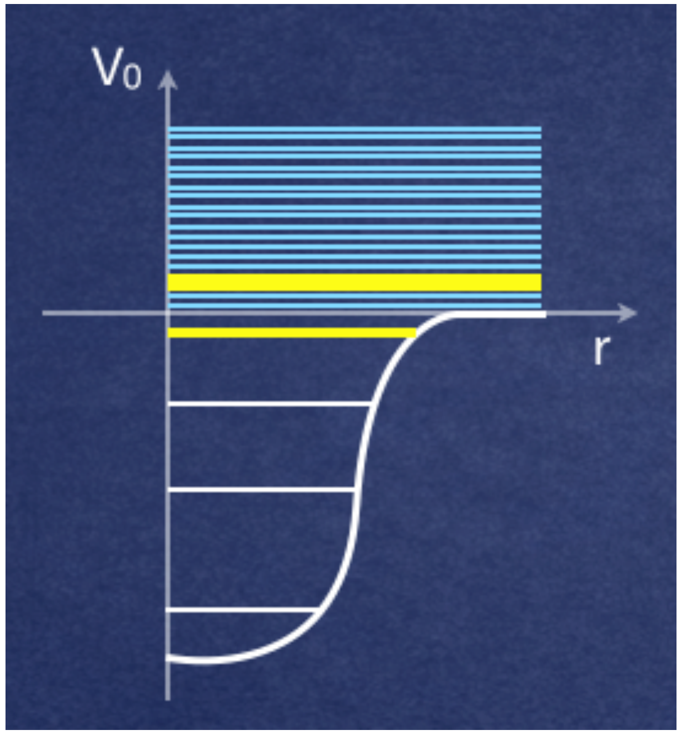
- **Open-quantum system extension** of the traditional Shell Model

- HO $\rightarrow$ **finite-depth** potential:
  - Woods-Saxon
  - Gamow Hartree Fock

- S.p. states, solutions of the one-body radial Schrödinger equation:

$$u''_{\alpha\ell j}(r) = \left[\frac{\ell(\ell+1)}{r^2} + \frac{2\mu}{\hbar^2}U(r) - k^2\right] u_{\alpha\ell j}(r)$$

$$U(r) = V_{WS}(r) + V_{so}(r)\vec{\ell}\cdot\vec{s} + U_C(r)$$



- **Specific boundary conditions**:
  - **Bound states, resonances**: $u_{n\ell j}(r) \sim C_+ H^+_\ell(\eta, kr), \quad r \rightarrow +\infty$
  - **Scattering states (continuum)**: $u_{k\ell j}(r) \sim C_+ H^+_\ell(\eta, kr) + C_- H^-_\ell(\eta, kr), \quad r \rightarrow +\infty$

The Framework: The Gamow Shell Model

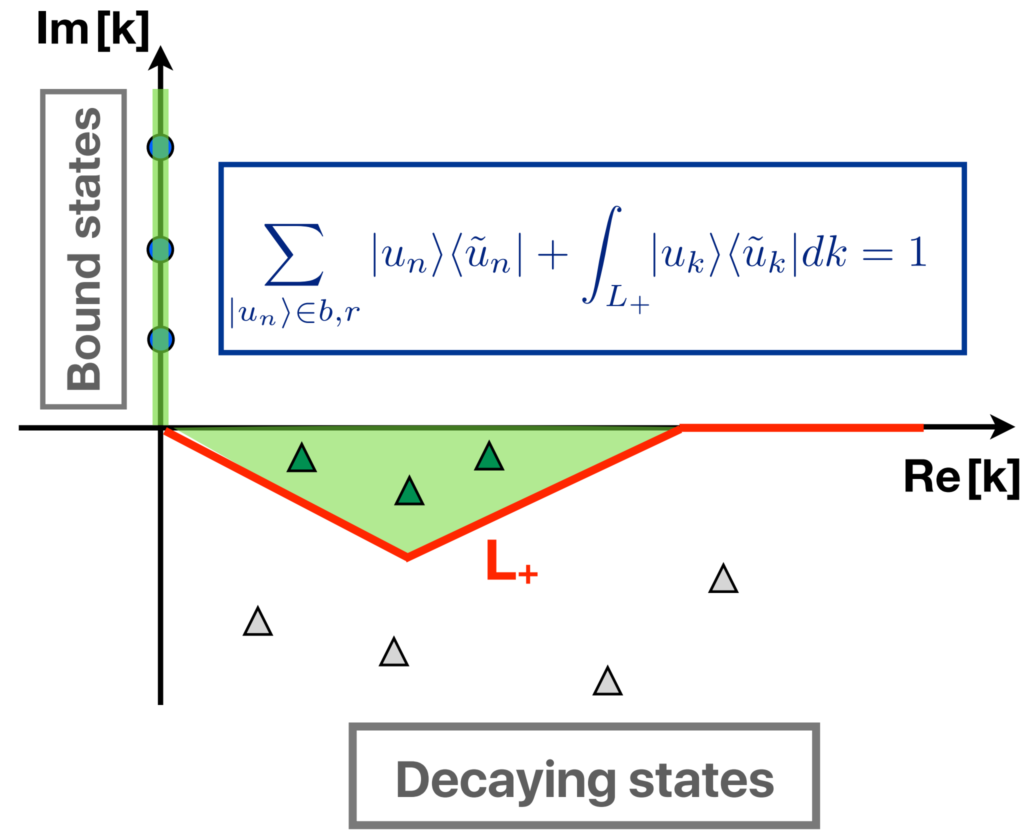
N. Michel, W. Nazarewicz, M. Płoszajczak, T. Vertse, J. Phys. G: Nucl. Part. Phys. 36, 013101 (2009)

- Both **correlations** and **continuum effects** are treated on the same footing (**Berggren** ensemble)

- Discretization of the contours → large basis

- GSM-Cluster orbital shell model (COSM) Hamiltonian:

$$H = \sum_{i=1}^{N_v} \left[\frac{\vec{p}_i^{\,2}}{2\mu_i} + U(i) \right] + \sum_{i<j=1}^{N_v} \left[V_{res}(i,j) + \frac{\vec{p}_i \cdot \vec{p}_j}{M_c} \right]$$



T. Berggren, Nucl. Phys. A 109, 265 (1968)

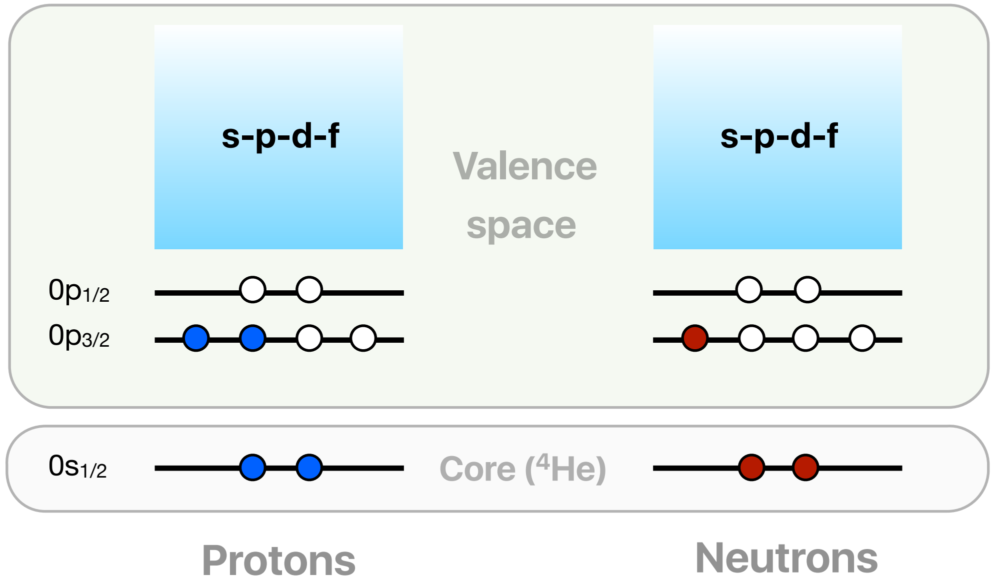
- Translational invariance, but approximate antisymmetry in the laboratory frame
- Exact treatment of the Coulomb interaction

- Diagonalization of *H* gives the A-body bound states + resonances

The Interaction for p-nuclei

Y. Jaganathen, R. Id Betan, N. Michel, W. Nazarewicz, M. Płoszajczak, Phys. Rev. C 96, 054316 (2017)

- **^{4}He core** modeled by a Woods-Saxon + spin-orbit + Coulomb



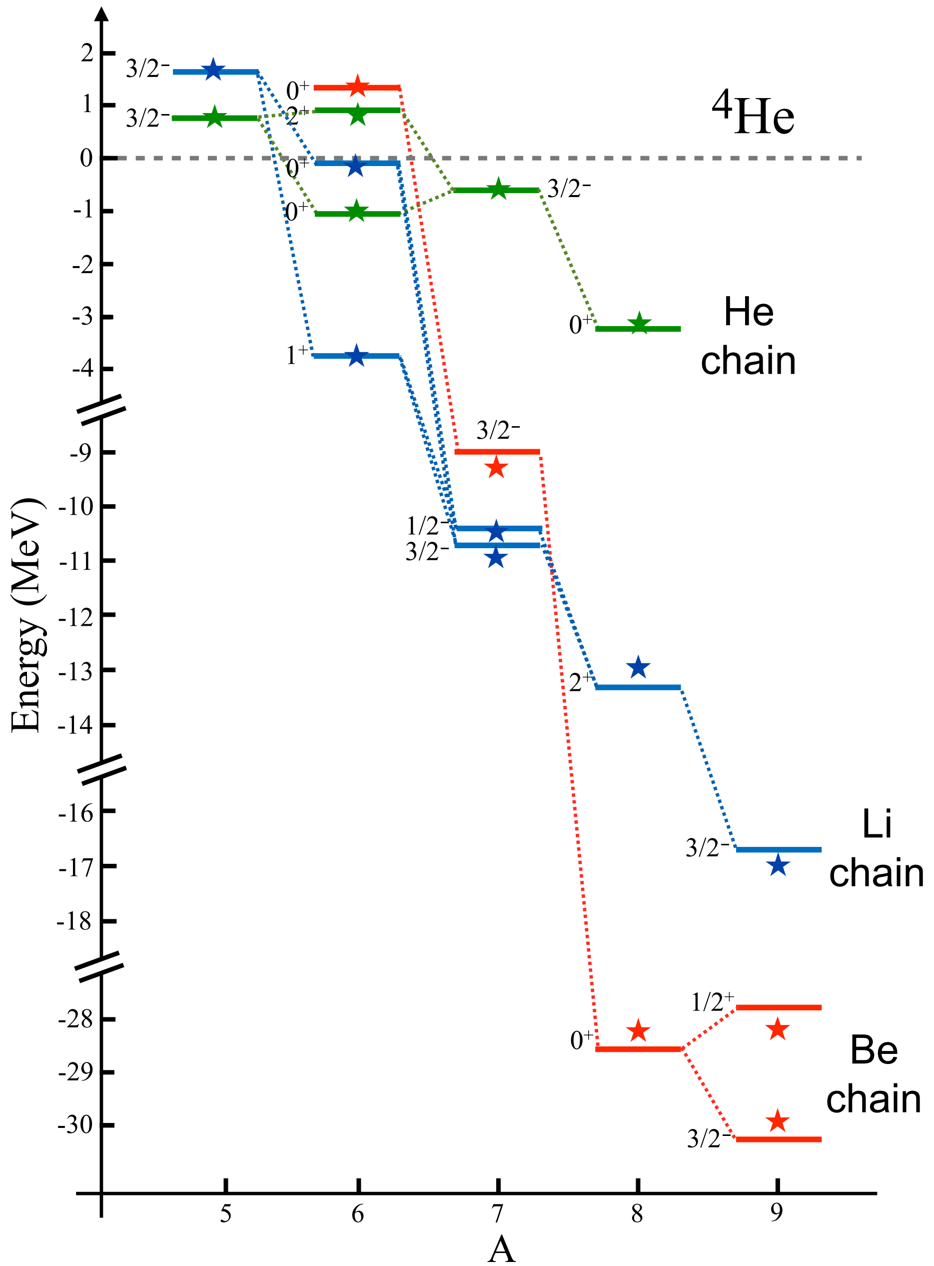
- **Configuration space: psdf**
  - $0p_{3/2}$, $0p_{1/2}$ and/or $1s_{1/2}$, $0d_{5/2}$ resonances
  - s, p, d and f scattering continua, $k_{max} = 2.0\ \mathrm{fm}^{-1}$
  - 4 nucleons in the continuum

- Effective finite-range NN potential
  - Gaussian-like with central + spin-orbit + tensor + Coulomb channels
  - Based on H. Furutani, H. Horiuchi, and R. Tamagaki, Prog. Theor. Phys. 62, 981 (1979)
  - **7 parameters** adjusted to the He, Li, Be chain ground-state energies + chosen excited states

- Statistical study to compute **uncertainties** and **correlations**

The zeroth order NN potential

Y. Jaganathen, R. Id Betan, N. Michel, W. Nazarewicz, M. Płoszajczak,
Phys. Rev. C 96, 054316 (2017)



Nucleus	State	E	$E_{\rm exp}$	Γ	$\Gamma_{\rm exp}$
^{6}He	0^+	-1.063	-0.973		
^{6}He	2^+	0.938	0.824	168	113(20)
^{7}He	$3/2^-$	-0.578	-0.528	178	150(20)
^{8}He	0^+	-3.225	-3.112		
^{6}Li	1^+	-3.724	-3.699		
^{6}Li	0^+	-0.054	-0.136		
^{7}Li	$3/2^-$	-10.688	-10.949		
^{7}Li	$1/2^-$	-10.359	-10.471		
^{8}Li	2^+	-13.350	-12.982		
^{9}Li	$3/2^-$	-16.677	-17.046		
^{6}Be	0^+	1.390	1.371	21	92(6)
^{7}Be	$3/2^-$	-8.977	-9.305		
^{8}Be	0^+	-28.572	-28.204	0	0.0056(3)
^{9}Be	$3/2^-$	-30.230	-29.870		
^{9}Be	$1/2^+$	-27.747	-28.186	0	217(10)

- s, p, d, f shells
- **4 nucleons in the continuum** (converged calculations)

- r.m.s. deviation of 250 keV
- Good starting point for detailed structural and reaction studies

The zeroth order NN potential

- Parameters

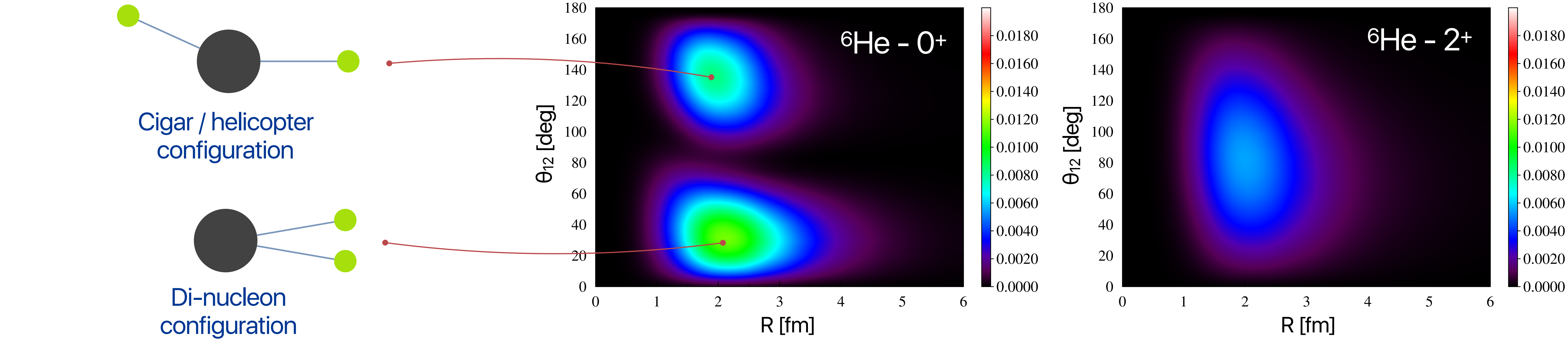
Parameter		Value
central	S=1 , T=1	-3.2 ± 22.0
	S=1 , T=0	**-5.1 ± 1.0**
	S=0 , T=0	**-21.3 ± 6.6**
	S=0 , T=1	**-5.6 ± 0.5**
spin-orbit	S=1 , T=1	-540 ± 1240
tensor	S=1 , T=1	-12.1 ± 79.5
	S=1 , T=0	**-14.2 ± 7.1**

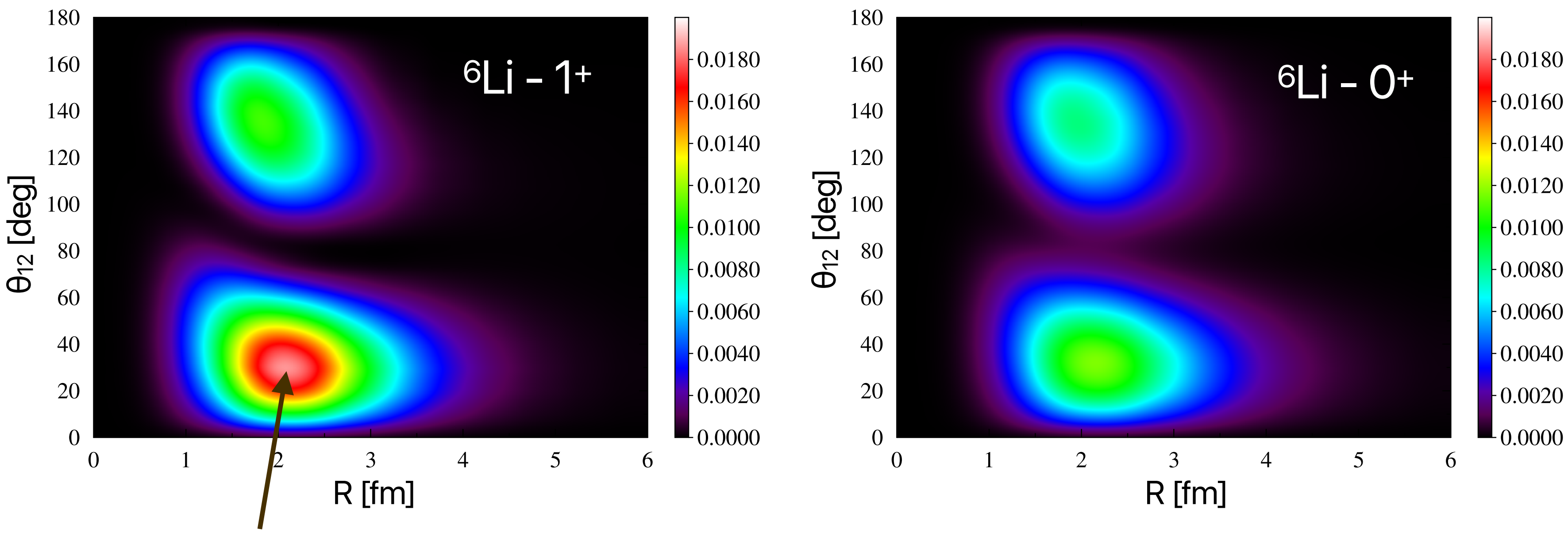
- Singular values (eigenvalues of the normalized Hessian matrix)

n	s_n	V_c^{11}	V_c^{10}	V_c^{00}	V_c^{01}	V_{LS}^{11}	V_T^{11}	V_T^{10}
1	243	0.00	**0.82**	−0.03	**0.53**	0.00	0.00	0.23
2	43.0	0.00	**−0.49**	−0.02	**0.85**	0.00	−0.01	−0.19
3	7.06	−0.04	−0.16	**0.79**	0.05	0.04	−0.07	**0.58**
4	3.94	0.02	−0.25	**−0.61**	0.01	−0.09	−0.04	**0.75**
5	0.57	−0.23	−0.02	−0.09	0.00	**0.97**	−0.01	0.04
6	0.20	**0.65**	−0.03	0.04	0.01	0.16	**0.74**	0.06
7	0.12	**0.73**	0.01	0.00	0.00	0.16	**−0.66**	−0.04

- Four parameters completely govern the optimization!
- The three remaining parameters are **sloppy**, i.e. unconstrained by the chosen set of experimental data

Applications - Correlation Densities





Strong T=0 deuteron correlation/clusterization

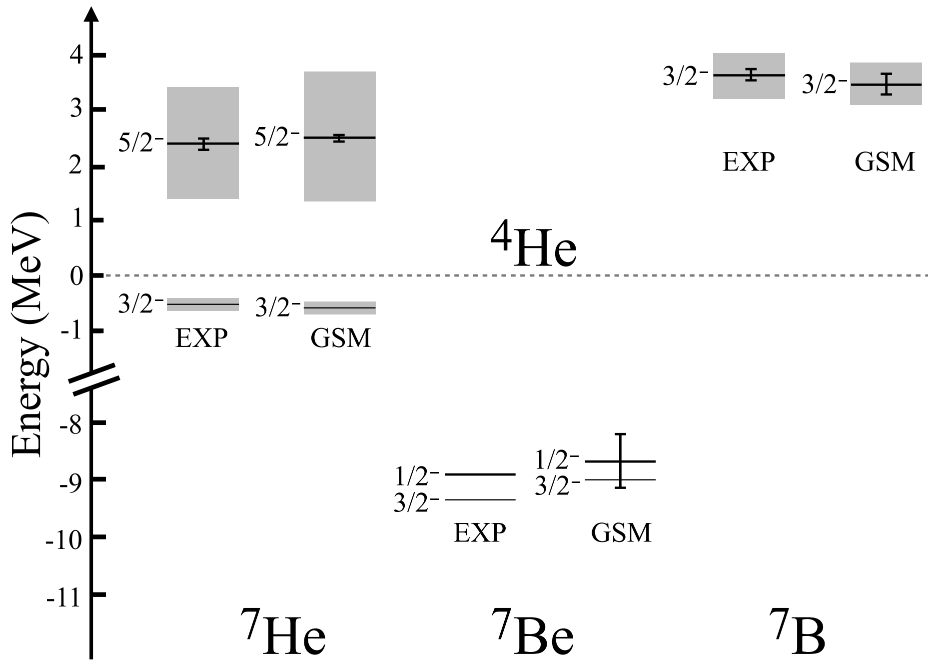
For further discussion:

K. Hagino and H. Sagawa, Phys. Rev. C **72**, 044321 (2005)

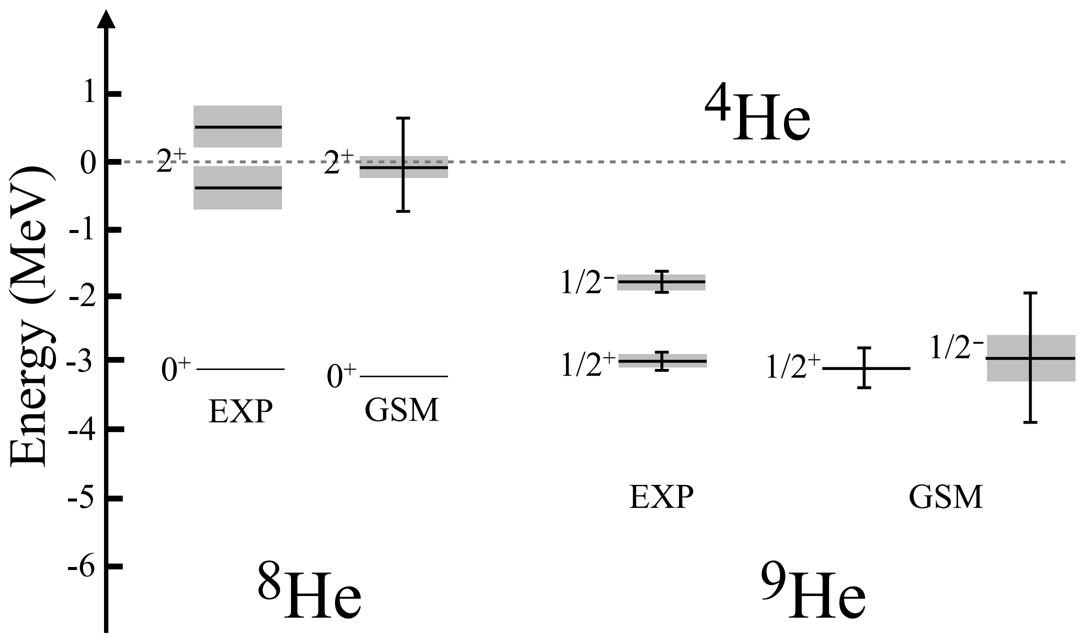
G. Papadimitriou, A. T. Kruppa, N. Michel, W. Nazarewicz, M. Płoszajczak and J. Rotureau Phys. Rev. C **84**, 051304(R) (2011)

Predictions - Energy Spectra with uncertainties

- **A=7 nuclei:**



- **Helium chain**



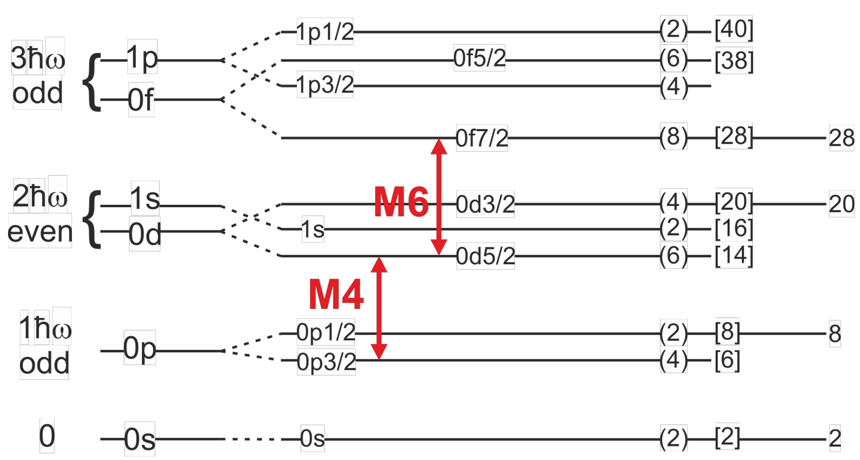
- Good overall agreement for the energies and the widths for the A=7 nuclei,
- But **large uncertainties** in the He chain which come from the sloppiness of the T=1 parameters.

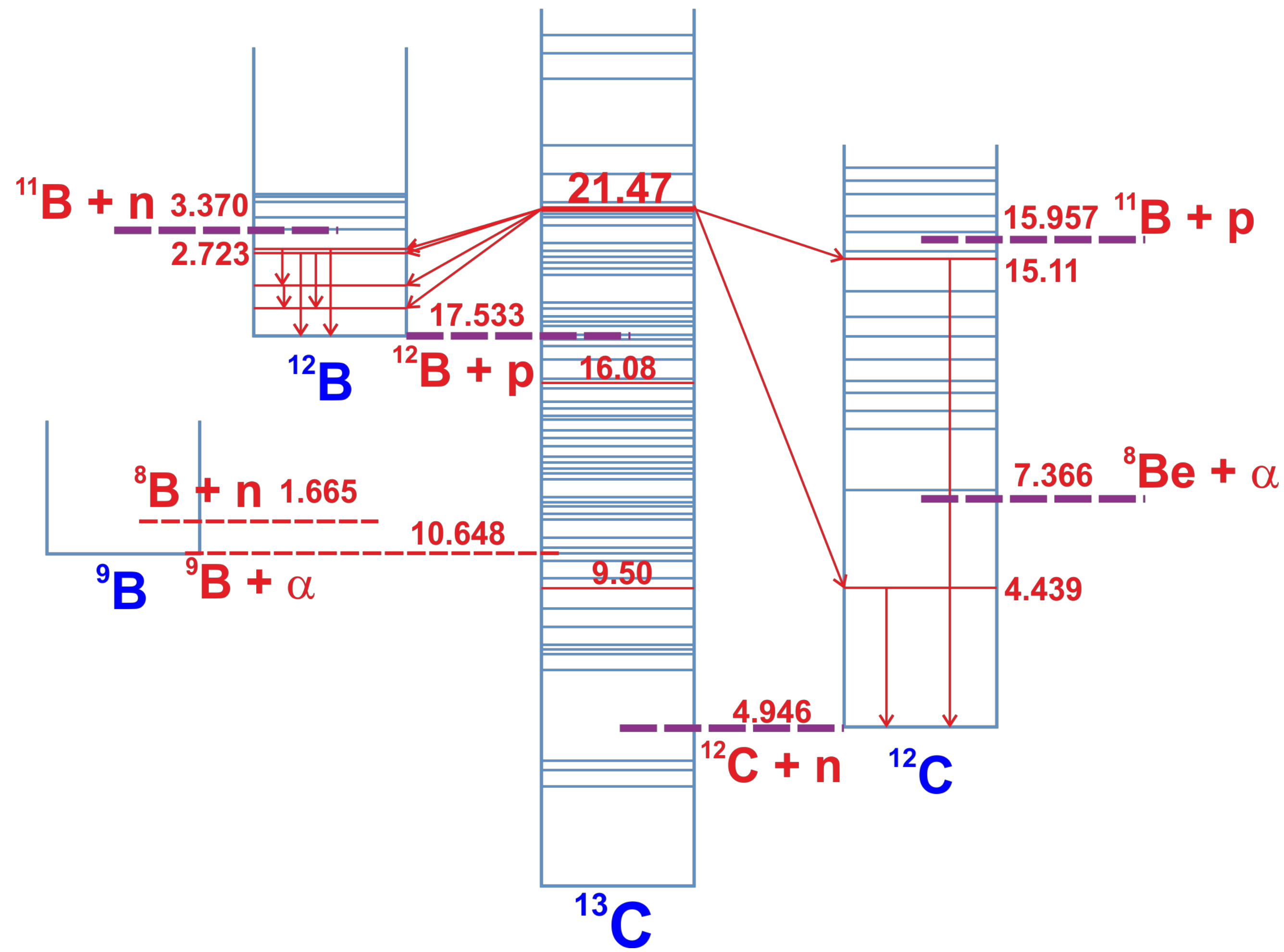
How to improve the interaction?

- How can we better constrain the parameters / add more correlations?
  - **Experimental data of different kinds**:
    - charge/matter radii,
    - EM moments...
  - Energies of **high-lying states** which:
    - can be described in the valence space
    - + are **simple** (from a SM point of view)
    - and not too embedded in the continuum (**close to particle emission thresholds**).

The 21.47-MeV M4 resonant excitation in ^{13}C

- Super-pure **stretched** coupling between $0p_{3/2}$ and $0d_{5/2}$ (ongoing theoretical analysis):





- See N. Cieplicka-Oryńczak's talk tomorrow on the proposed experiment at the Cyclotron Centre Bronowice (CCB) in Kraków.

(Picture courtesy of N. Cieplicka-Oryńczak)

Summary and Outlook

- Within the Gamow Shell Model, we currently have:
  - a well-optimized **interaction** for p-nuclei
  - a code which can calculate all **observables** (within the GSM framework).

- To improve the interaction, we need for **exotic states**:
  - experimental data of different kinds (**charge/matter radii, EM moments**...)
  - **stretched** / simple high-lying states.

- Outlook: a GSM interaction for sd-nuclei (A > 16):
  - Local interaction for small chains of nuclei (on-going collaboration with the Milano group on the O chain, and possible collaboration with the Legnaro group)
  - Global interaction, if there is a high interest...

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- Within the Gamow Shell Model, we currently have:
  - a well-optimized **interaction** for p-nuclei
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Thank you for your attention!

Back-up

$$V = V_c + V_{LS} + V_T + V_{\text{Coul}}.$$

$$\tilde{V}_c(r) = \sum_{n=1}^{3} V_c^n \left(W_c^n + B_c^n P_\sigma - H_c^n P_\tau - M_c^n P_\sigma P_\tau\right) e^{-\beta_c^n r^2} \tag{5}$$

$$\tilde{V}_{LS}(r) = \boldsymbol{L}\cdot\boldsymbol{S} \sum_{n=1}^{2} V_{LS}^n \left(W_{LS}^n - H_{LS}^n P_\tau\right) e^{-\beta_{LS}^n r^2} \tag{6}$$

$$\tilde{V}_T(r) = S_{ij} \sum_{n=1}^{3} V_T^n \left(W_T^n - H_T^n P_\tau\right) r^2 e^{-\beta_T^n r^2}, \tag{7}$$

where $r \equiv r_{ij}$ stands for the distance between the nucleons i and j, $\boldsymbol{L}$ is the relative orbital angular momentum, $\boldsymbol{S} = (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)/2$, $S_{ij} = 3(\boldsymbol{\sigma}_i \cdot \hat{r})(\boldsymbol{\sigma}_j \cdot \hat{r}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$, and P_σ and P_τ are spin and isospin exchange operators, respec-

In order to be applied in the present GSM formalism, the interaction is rewritten in terms of the spin-isospin projectors Π_{ST} [51, 52]:

$$V_c(r) = V_c^{11} f_c^{11}(r)\Pi_{11} + V_c^{10} f_c^{10}(r)\Pi_{10} + V_c^{00} f_c^{00}(r)\Pi_{00} + V_c^{01} f_c^{01}(r)\Pi_{01}, \tag{8}$$

$$V_{LS}(r) = (\boldsymbol{L}\cdot\boldsymbol{S})\, V_{LS}^{11} f_{LS}^{11}(r)\Pi_{11}, \tag{9}$$

$$V_T(r) = S_{ij}\left[V_T^{11} f_T^{11}(r)\Pi_{11} + V_T^{10} f_T^{10}(r)\Pi_{10}\right], \tag{10}$$