

## MODEL AND RESULTS

We examine the shape phase transitions and critical point states along  $Z = 114$  and  $Z = 120$  long isotopic chains applying the deformation-constrained Skyrme-Hartree-Fock-Bogoliubov model. The self-consistent Skyrme-HFB approach is equivalent to minimization of the total energy  $E^{tot}$  of Skyrme energy density functional under constraints of average values of proton/neutron numbers  $\langle \hat{N}_{p/n} \rangle = N_{p/n}$  and of multipole moments  $\langle \hat{Q}_{\lambda\mu} \rangle = Q_{\lambda\mu}$ , see eg. Ref. [1]. For the mass multiple moment operators, we use two constrained values of quadrupole moments  $\hat{Q}_{20}$  and  $\hat{Q}_{22}$ , but our results are presented using the Bohr deformation parameters  $\beta$  and  $\gamma$  which are extracted via relations:

$$\beta = \sqrt{\frac{5}{16\pi} \frac{4\pi}{3AR_0^2} \sqrt{Q_{20}^2 + 2Q_{22}^2}} = \frac{\sqrt{5\pi}}{3r_0^2 A^{5/3}} \sqrt{Q_{20}^2 + 2Q_{22}^2},$$

$$\gamma = \tan^{-1} \frac{\sqrt{2}Q_{22}}{Q_{20}}.$$

The above equality constraint problem is resolved self-consistently (by iteration until convergence) using an augmented Lagrangian method [2] with the symmetry-unrestricted code HFODD [3]. In the particle-hole channel the Skyrme SkM\* force [4] was applied and a density-dependent *mixed* pairing [1, 5] interaction in the particle-particle channel was used. The code HFODD uses a three-dimensional Cartesian deformed harmonic oscillator basis. In the present study, the basis was composed of the 1140 lowest states taken from 26 harmonic oscillator shells.

In order to describe the shape phase transitions and critical point symmetries in nuclei we would like to use the interacting boson model (IBM) Ref. [6] benchmarks of collective behaviour with three groups of dynamical symmetries: U(5), SU(3)/SU(3) and O(6), which represents spherical ground state, axially symmetric prolate/oblate deformations and  $\gamma$ -soft nuclei, respectively. There are also two critical point solutions: X(5) the first-order phase transition between spherical, U(5), nuclei and axially deformed SU(3)/SU(3) nuclei, and E(5) which is the second-order phase transition between spherical, U(5), nuclei and  $\gamma$ -soft, O(6), nuclei, see Refs. [7-9].

In Figs. 1 and 2 we present our results for the  $Z = 114$  and  $Z = 120$  isotopic chains as a two-dimensional  $\beta$ - $\gamma$  maps with marked positions of the ground states, saddle points and the inner fission barrier heights  $B_f$ . For  $Z = 120$  isotopes, in Fig. 2, we have also marked the so-called *superdeformed oblate* (SDO) minima [10].

### $Z = 114$

The isotopes with  $N = 154$ -168 have the axially symmetric prolate-deformed ground states, so they belong to SU(3) IBM dynamical symmetry group. All of them have an axially symmetric saddle point but the last three nuclei have also a triaxial saddle point. For isotopes with  $N = 170$ -178 there is visible competition between the prolate and oblate minima and these transitional  $\gamma$ -soft nuclei are classified to the O(6) dynamical symmetry group. For nucleus  $^{294}\text{Fl}$  ( $N = 180$ ), we have the second-order phase transition E(5), which is the critical point solution for this examined isotopic chain. The next six nuclei (with  $N = 182$ -192) have the spherical ground state and these isotopes are the members of U(5) dynamical symmetry group, but for nucleus with  $N = 192$  neutrons exists the competition between spherical and triaxial ground state. The isotopes with  $N = 194$  and  $196$  neutrons have only the triaxial ground state.

For all of even-even  $Z = 114$  isotopes, we can see that with the increasing number of neutrons, the fission barrier height also increases as to with  $N = 180$ , where the fission barrier achieves the highest value, equals 6.72 MeV. Then, barrier starts decrease.

### $Z = 120$

The nuclei with  $N = 160$  and  $162$  neutrons have the prolate-deformed ground state (SU(3) dynamical symmetry group), but there also exists the SDO minimum, which becomes the local ground state for isotopes with  $N = 164$  and  $166$  neutrons. In the isotopes with  $N = 168$ -178 the ground states are oblate-deformed, but there are also the second local prolate minima. These two minima are connected to each other via triaxial  $\gamma$ -degree of freedom and all those isotopes belong to the O(6) dynamical symmetry group. In an isotope with neutron number  $N = 180$  the barrier between these two minima disappears and this isotope acquires the critical point symmetry E(5). All other isotopes with  $N = 182$ -194, except the last one ( $N = 194$ ) with the triaxial ground state, are spherical in the ground state and they are classified to the U(5) group.

For all of  $Z = 120$  isotopes, we can see that the fission barrier height increases with increasing number of neutrons, as to nucleus with  $N = 182$ , where the fission barrier is the highest and equals 8.21 MeV.

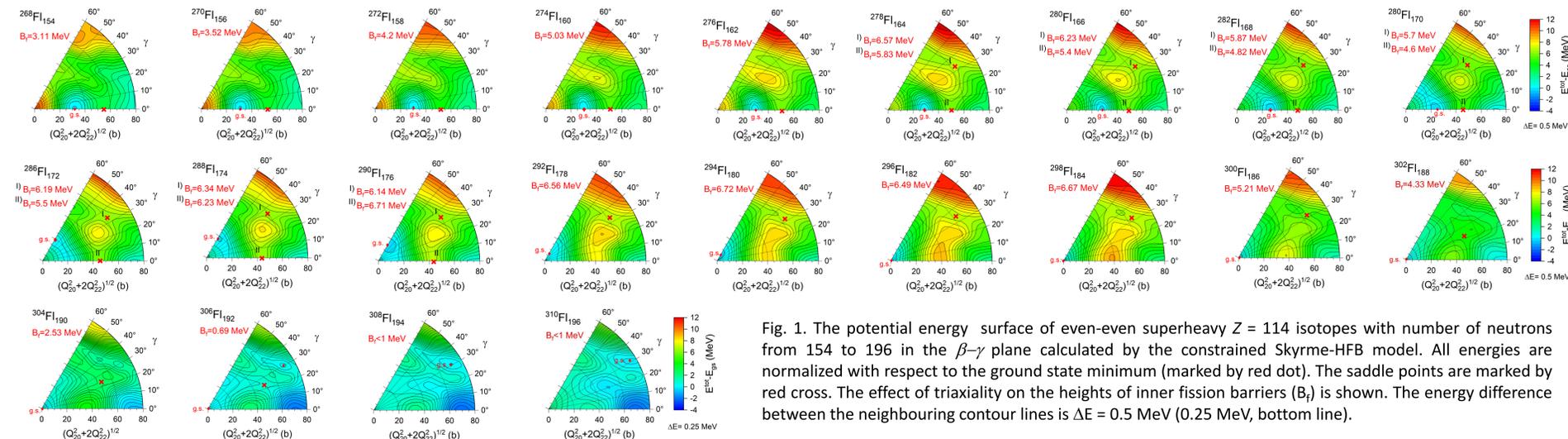


Fig. 1. The potential energy surface of even-even superheavy  $Z = 114$  isotopes with number of neutrons from 154 to 196 in the  $\beta$ - $\gamma$  plane calculated by the constrained Skyrme-HFB model. All energies are normalized with respect to the ground state minimum (marked by red dot). The saddle points are marked by red cross. The effect of triaxiality on the heights of inner fission barriers ( $B_f$ ) is shown. The energy difference between the neighbouring contour lines is  $\Delta E = 0.5$  MeV (0.25 MeV, bottom line).

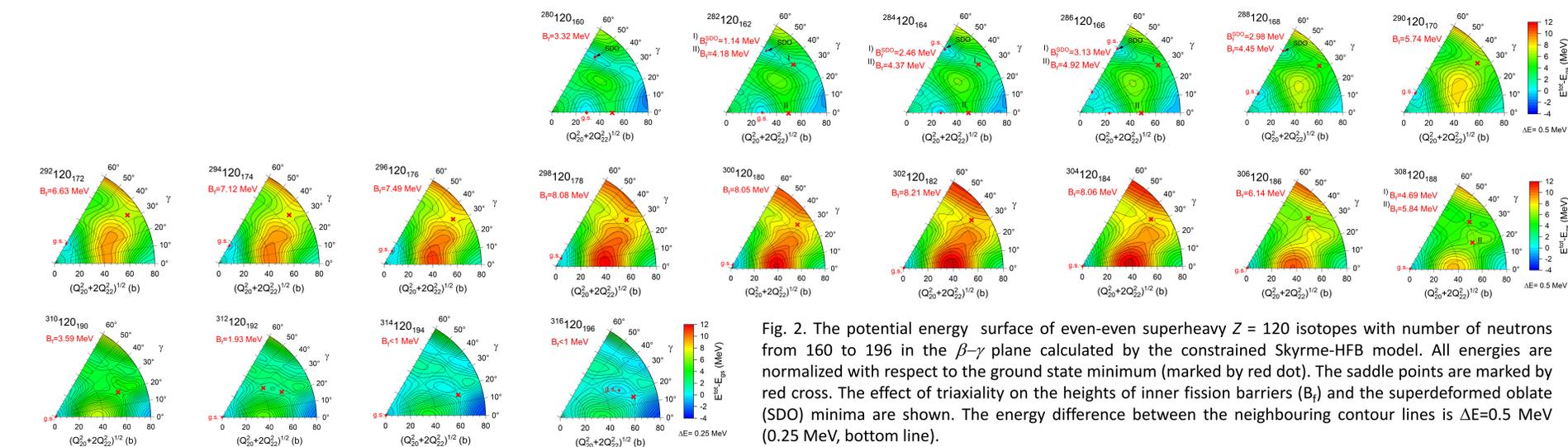


Fig. 2. The potential energy surface of even-even superheavy  $Z = 120$  isotopes with number of neutrons from 160 to 196 in the  $\beta$ - $\gamma$  plane calculated by the constrained Skyrme-HFB model. All energies are normalized with respect to the ground state minimum (marked by red dot). The saddle points are marked by red cross. The effect of triaxiality on the heights of inner fission barriers ( $B_f$ ) and the superdeformed oblate (SDO) minima are shown. The energy difference between the neighbouring contour lines is  $\Delta E = 0.5$  MeV (0.25 MeV, bottom line).

## CONCLUSIONS

- The even-even superheavy isotopes  $Z=114$  form three regions: prolate-deformed SU(3) (for  $154 \leq N \leq 168$ ), spherical U(5) (for  $N > 180$ ), and transitional region ( $\gamma$ -soft) O(6) between the former two.
- The even-even superheavy isotopes  $Z=120$  form three regions: oblate-deformed SU(3) (for  $160 \leq N \leq 168$ ), spherical U(5) (for  $N > 180$ ), and transitional region ( $\gamma$ -soft) O(6) between the former two.
- On the border between the O(6) and U(5) regions (for  $N = 180$ ) nuclei exhibit a rather flat potential bottom and acquire the critical point symmetry E(5).
- For  $Z = 114$  isotopes the fission barrier is the highest for  $N = 180$  and equals 6.72 MeV.
- For  $Z = 120$  isotopes the fission barrier achieves the highest value equals 8.21 MeV for nucleus with  $N = 182$  number of neutrons.
- Two-dimensional  $\beta$ - $\gamma$  analysis allows reduction of the fission barriers of Ref. [5] by  $\sim 1$  MeV for  $Z = 114$  isotopes and by  $\sim 2$  MeV for  $Z = 120$  isotopes.

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