Structure and reactions of one-neutron halo nuclei

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The critical description of the experimental results from complementary approaches could be of extreme interest.

Can Ab Initio Theory Explain the Phenomenon of Parity Inversion in $^{11}$Be?

**TRIUMF**

$^{11}$Be within NCSMC: Discrimination among chiral nuclear forces

![Diagram showing the energy levels and parity inversion in $^{11}$Be](image)
Can we obtain an alternative description in terms of elementary modes of excitation?

Independent Particles

Collective Phonons

Particle-vibration coupling

Hartree-Fock mean Field

Random Phase Approximation
Parity inversion in N=7 isotones is not reproduced by spherical mean field calculations

Typical spherical mean-field results with Skyrme forces
(Sagawa,Brown,Esbensen PLB 309(93)1)
A possible explanation of parity inversion: dynamical coupling between the core and the loosely bound neutron

The core: spherical or deformed?
Important role of fluctuations expected
We propose a dynamical description

Myo et al, PRC 86 (2012) 024318
The admixture of $d_{5/2} \times 2^+$ configuration in the $1/2^+$ g.s. of $^{11}\text{Be}$ is about 15%.

$^{9}\text{Be}(^{11}\text{Be},^{10}\text{Be}+\gamma)X$

$^{p}(^{11}\text{Be},^{10}\text{Be})d$

T. Aumann et al. PRL 84 (2000) 35


Basic effect of particle-vibration coupling on the single-particle energies close to the Fermi energy

\[ L = \frac{h^2(j,j',L)}{e_j - (e_{j'} + \hbar \omega_\lambda)} < 0 \]

\[ + \]

\[ h^2(j,j',L) = \frac{(e_j - e_{j''} + \hbar \omega_\lambda)}{\omega_{\lambda}} > 0 \]

From B(EL) experimental value

\[ h(a, b\lambda) = \frac{1}{\sqrt{4\pi}} \langle j_a \lambda | j_b \rangle \beta_\lambda \left( j_a \left| \frac{\partial U}{\partial r} \right| j_b \right) \]
$^{11}\text{Be}$

$E_{\text{shift}} = -2.5 \text{ MeV}$

$s_{1/2}$

$d_{5/2}$

$s_{1/2}$

$2^+$

$E_{\text{shift}} = +2.5 \text{ MeV}$

Self-energy

Pauli blocking of core ground state correlations

Level inversion
**Ingredients of our calculation**

\[
B(E2) = 10.4 \pm 1.2 \text{ e}^2 \text{ fm}^4
\]

\[
\beta_{em} = 1.12 \quad \beta_n \approx 0.9
\]

**Fermionic degrees of freedom:**

- \(s_{1/2}, p_{1/2}, p_{3/2}, d_{5/2}\) Wood-Saxon levels in a box

**Bosonic degrees of freedom:**

- \(2^+, 3^-\), pair vib. QRPA solutions tuned to reproduce available exp. data

\[\text{(Saxon - Woods + spin - orbit)}\]
We perform the many-body calculation starting from a Woods-Saxon potential, with a spatially dependent effective mass, with

\[ m_k(r=0) = 0.7 \text{ m}, \ m_k(r \gg R) = \mu = 0.91 \text{ m} \]

The following parameters are fitted to obtain the best agreement of the renormalized energies with the experimental \( \frac{1}{2}^+, \frac{1}{2}^- \) and \( \frac{5}{2}^+ \) states in \(^{11}\text{Be}\) and \( \frac{3}{2}^- \) in \(^{9}\text{Be}\):

- Depth, diffuseness, radius, strength of spin-orbit coupling
\[\sqrt{0.83}|s_{1/2} > + \sqrt{0.17}|(d_{5/2} \otimes 2^+)_{1/2} >\]

\[\sqrt{0.81}|p_{1/2} > + \sqrt{0.02}|(d_{5/2} \otimes 3^-)_{1/2} > + \sqrt{0.15}|((p_{1/2}, 1p_{3/2}^-)_{2+} \otimes 2^+) p_{1/2} >\]

\[\sqrt{0.34}|d_{5/2} > + \sqrt{0.32}|(s_{1/2} \otimes 2^+)_{5/2} > + \sqrt{0.34}|(d_{5/2} \otimes 2^+)_{5/2} >\]
Strength of the dipole transition between $\frac{1}{2}+$ and $\frac{1}{2}-$ states

$M(E1) X$

1.95 e fm

$-0.26$ e fm

$-0.19$ e fm

$B(E1) (\text{th.}) = 0.11$ e$^2$ fm$^2$

$B(E1) (\text{exp.}) = 0.102 \pm 0.002$ e$^2$ fm$^2$

This result is sensitive to the details of the mean field potential
Isotopic shift of the charge radius

\[
\langle r^2 \rangle_{10\text{Be}}^{\frac{1}{2}} = 2.361 \pm 0.017 \text{ fm} \quad \langle r^2 \rangle_{11\text{Be}}^{\frac{1}{2}} = 2.466 \pm 0.015 \text{ fm}
\]

Single-particle picture: \( S=1 \)

Many-body picture: \( S=0.83 \)

\[
\langle r^2 \rangle_{11\text{Be}} = \left( \langle r^2 \rangle_{10\text{Be}} + \frac{\langle r^2 \rangle_{1s1/2}^{1/2}}{11} \right)^2 \times S^2 + (1 - S^2) \times \left( \langle r^2 \rangle_{10\text{Be}} \left( 1 + \frac{2}{4\pi} \beta^2 \right) + \frac{\langle r^2 \rangle_{d5/2,\text{coll}}^{1/2}}{11} \right)^2 = .
\]

\[
\langle r^2 \rangle_{10\text{Be}} + \frac{\langle r^2 \rangle_{1s1/2}^{1/2}}{11} \times S^2 + (1 - S^2) \times \left( \frac{\langle r^2 \rangle_{d5/2,\text{coll}}^{1/2}}{11} \right)^2 + \frac{\langle r^2 \rangle_{10\text{Be}}}{4\pi} \beta^2 \left( 2 \right)
\]

\( \Delta \langle r^2 \rangle_{11\text{Be}}^{\frac{1}{2}} \text{ (th.)} = 0.12 \text{ fm} / 0.27 \text{ fm} \)

\( \Delta \langle r^2 \rangle_{11\text{Be}}^{\frac{1}{2}} \text{ (exp.)} = 0.11 \text{ fm} \)
Matrix elements due to GSC Pauli rearrangement

The contribution of a given p-h configuration to the GS Correlation Energy is (B&Mill)

\[ \delta E = \frac{-\hbar_{ai,bk,\lambda} \sqrt{2j_a+1}}{0 - (E_{ai} + E_{bk} + \hbar \omega_{\lambda})} < 0 \]

The presence of a new neutron (scattering- or bound-like) inhibits some of these correlations, producing an energy modification of the core state...

\[ -\frac{\delta E}{2j_a+1} > 0 \]

This is the meaning/value of the NFT self energy diagram (B&Mill, eq.6.225)

\[ \frac{-1}{E_{ai} - (2E_{ai} + E_{bk} + \hbar \omega_{\lambda})} \left\langle \left( (j_{a1}, j_{a2}) J = 0, j_{a3} ; j_a \right| (j_{a1}, j_{a3}) J = 0, j_{a2} ; j_a ) \right\rangle \]

\[ = \frac{(\hbar_{ai,bk,\lambda})^2}{(E_{ai} + E_{bk} + \hbar \omega_{\lambda})} \]
One more auxiliary GSCPR channel.

\[ \psi_{jama} = [\psi_{jama}^x + \psi_{jama}^C \Gamma^{\lambda+}] \psi_{jama} \Phi_{GS} \]

**but if**

\[ \psi_{\text{GS}} = [1 + \epsilon \psi_{j_{\text{a,occ}}}^{-1} \psi_{j_{\text{b}}}^{x} \Gamma^{\lambda+}]_{j_{\text{a}}} \psi_{j_{\text{a}}} \psi_{j_{\text{b}}}^{x} \Gamma^{\lambda+}]_{j_{\text{b}}} \psi_{j_{\text{b}}}^{x} \Gamma^{\lambda+}]_{j_{\text{c}}} + \cdots \Phi_{GS}^{HF} \]

\[ a \text{ new term } l \text{ must be added} \]

\[ \psi_a = [\psi_a^x + \psi_b^{D \Gamma^{\lambda+}}]_{j_{\text{a}}} \psi_a^{y} - [\psi_c^{D \Gamma^{\lambda+}}]_{j_{\text{c}}} + \cdots \Phi_{GS} \]

**with the hole anihilator**

\[ \psi_a^{y} = (\psi_a^{y}(r)/r) \Theta_{j_{\text{a}}} \]

\[ \psi_a^{y}(r) = \sum_i y_{ai} R_{ai}^{(r)} ; e_{ai} < e_F \]

**GSC Pauli rearr.:**

**An auxiliary Coupled Channel**

\[
\begin{pmatrix}
H_p - e_F & \Xi_{a,b,\lambda} f(r) & 0 & \Xi_{a,c,\lambda} f(r) & \Xi_{a,b,\lambda} f(r) & \Xi_{a,c,\lambda} f(r) \\
\Xi_{a,b,\lambda} f(r) & H_p - e_F + \hbar \omega & \Xi_{a,b,\lambda} f(r) & 0 & \Xi_{a,c,\lambda} f(r) & \Xi_{a,c,\lambda} f(r) \\
0 & \Xi_{a,b,\lambda} f(r) & (H_p - e_F) & -\Xi_{a,c,\lambda} f(r) & -\Xi_{a,c,\lambda} f(r) & -\Xi_{a,c,\lambda} f(r) \\
\Xi_{a,c,\lambda} f(r) & 0 & -\Xi_{a,c,\lambda} f(r) & (H_p - e_F) & -\Xi_{a,c,\lambda} f(r) & -\Xi_{a,c,\lambda} f(r)
\end{pmatrix}
\]

\[\begin{pmatrix}
u_a^x \\
u_b^x \\
u_c^x \\
u_b^y \\
u_a^y \\
u_c^y
\end{pmatrix} = \begin{pmatrix}
u_a^x \\
u_b^x \\
u_c^x \\
u_b^y \\
u_a^y \\
u_c^y
\end{pmatrix}
\]

Creation of a neutron in an "occupied" level \( j_{\text{a}} \) level may/will give a nonnull contribution

p-h + phonon "virtual" excitations with respect to the HF GS.
$^{10}$Be(d,p)$^{11}$Be at $E_d = 21.4$ MeV

Test of the single-particle component of the many-body wavefunction

Form factors

Cross sections

K.T. Schmitt et al., PRC88 (2012) 064612
\[ ^{11}\text{Be}(1/2^+) (p,d)^{10}\text{Be}(2^+) \]

Test of the collective component \( R^C_{d5/2} \) of the many-body wavefunction (but we should calculate the optical potential microscopically!)

\[ (\psi^C_b \otimes \Gamma^+_\chi)_{j_\alpha} = (R^C_b (r)/r)(\Theta_{j_b} \otimes \Gamma^+_\chi)_{j_\alpha} \]

d5/2 phase shift in the bare potential

Renormalized 5/2+ phase shift

\[ E_{\text{res}} = 6.5 \text{ MeV} \]

\[ E_{\text{res}} = 1.25 \text{ MeV} \]  
\[ \Gamma = 160 \text{ keV} \]
The usually quoted value of width of the 5/2+ resonance (100 keV) is derived from $^9\text{Be}(t,p)^{11}\text{Be}$ spectra

The width from $^{10}\text{Be}(d,p)^{11}\text{Be}(5/2^+)$ spectra is much larger and is well reproduced by theory.
It is possible to obtain a quantitative description of the structure and of the reactions of $^{11}\text{Be}$, based on the dynamical coupling of particles and vibrations, taking properly into account ground state correlations.

Extend the calculations and check theory in neighbouring nuclei $^{11}\text{N}, (^{10}\text{Li}, ^{12}\text{B}, ^{13}\text{C})$. 
Basic effect of particle-vibration coupling on the single-particle energies close to the Fermi energy:

\[
\begin{align*}
\text{(A)} & \quad L = \frac{h^2(j,j',L)}{e_j - (e_{j'} + \hbar \omega \lambda)} < 0 \\
\text{(B)} & \quad L = \frac{h^2(j,j',L)}{e_j - e_{j''} + \hbar \omega \lambda} > 0
\end{align*}
\]

This is a UNIVERSAL RESULT: Green’s function, Equations of Motion, in general any many-body theory based on single-particle picture.
V_0 = 70\text{MeV}
\quad a = 0.81\text{fm}
\quad R = 2.1\text{fm}

Two Phonon Anharmonicities

Butterfly Diagrams

\begin{align*}
E(\text{MeV})
\end{align*}

- 5/2^+: a |d5/2\rangle + b |s1/2\times2\rangle + c |d5/2\times2\rangle
- 1/2^+: A |s1/2\rangle + B |d5/2\times2\rangle
- 1/2^-: \alpha |p1/2\rangle + \beta |p3/2\times2\rangle

0th (Bare) 1st Rainbow 3rd

PAULI!

PAULI!
Microscopic description of superfluid nuclei beyond mean field: iterating the PVC with Nambu-Gor’kov formalism

by extending the Dyson equation…

\[ G^{-1}_\mu = (G^0_\mu)^{-1} - \Sigma_\mu (\omega) \]

… to the case of superfluid nuclei (Nambu-Gor’kov), it is possible to consider both:

F. Barranco et al., EPJ A21 (2004) 57
A. Idini et al., PRC 85 (2012) 014
cf. V. Soma’, C. Barbieri, T. Duguet,
PRC 84 (2011) 064317; PRC 87 (2013) 011303
Based on this approach, we could calculate several nuclear structure observables in $^{120}$Sn with a 10% error.

<table>
<thead>
<tr>
<th>Observables</th>
<th>Opt. levels</th>
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<tbody>
<tr>
<td>$\Delta$ (keV)</td>
<td>50 (3.5 %)</td>
</tr>
<tr>
<td>$E_{qp}$ (keV)</td>
<td>45 (4.5 %)</td>
</tr>
<tr>
<td>Mult. splitt. (keV)</td>
<td>59 (8.4 %)</td>
</tr>
<tr>
<td>$d_{5/2}$ (centr.) (keV)</td>
<td>40 (4%)</td>
</tr>
<tr>
<td>$d_{5/2}$ (width) (keV)</td>
<td>8 (1%)</td>
</tr>
<tr>
<td>$B(E2)/B_{sp}$</td>
<td>1.43 (14%)</td>
</tr>
<tr>
<td>$\sigma_{2n}(p,t)$ (mb)</td>
<td>40 (2%)</td>
</tr>
</tbody>
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