

Structure and reactions of one-neutron halo nuclei

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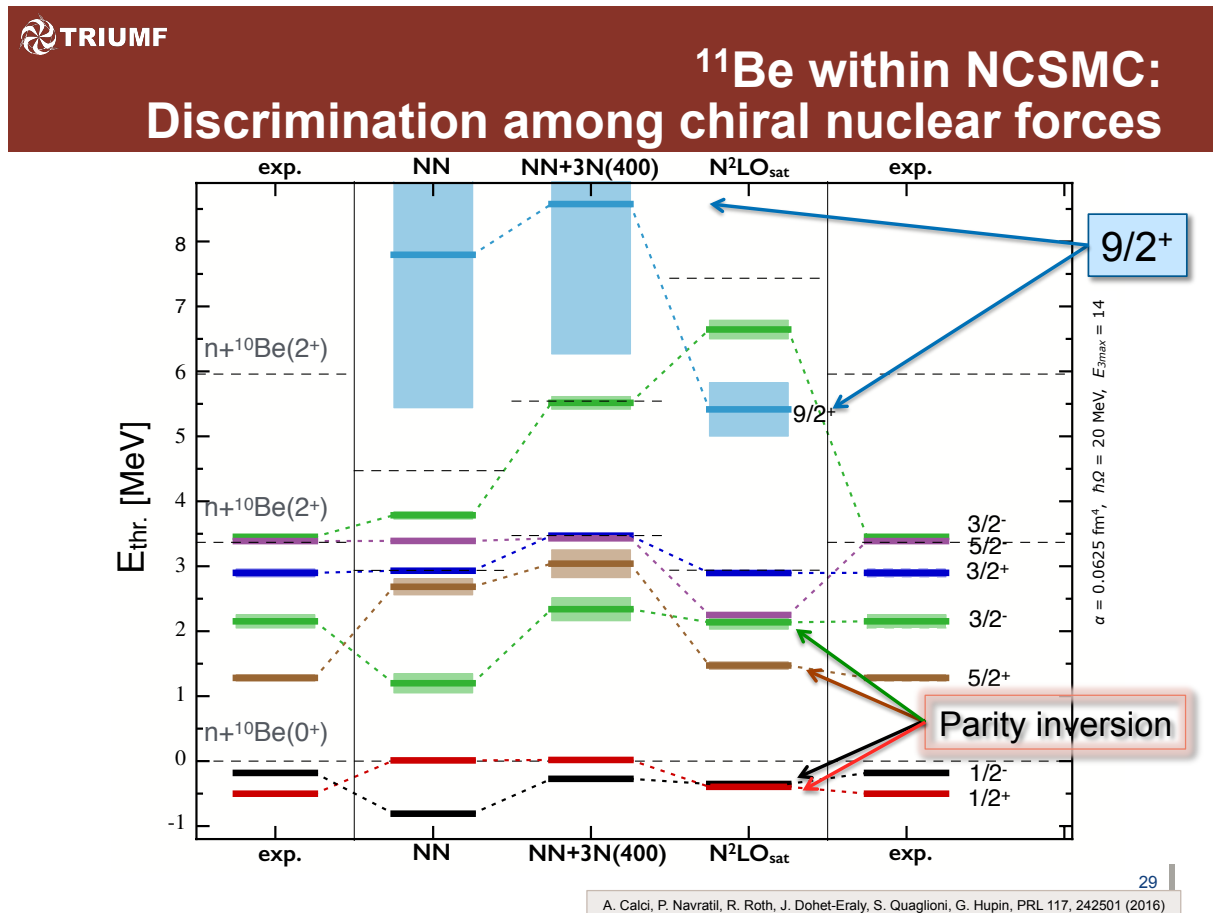
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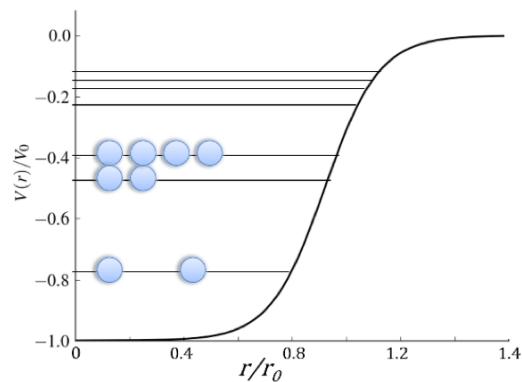
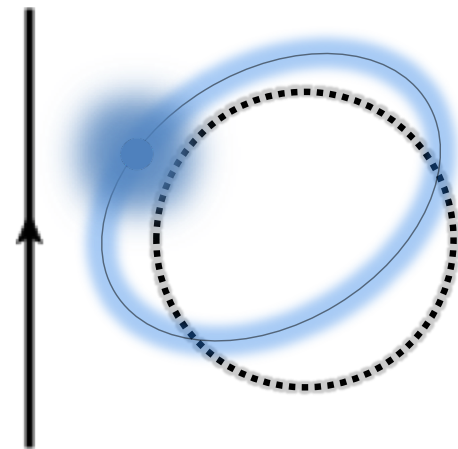
The critical description of the experimental results from complementary approaches could be of extreme interest

Can *Ab Initio* Theory Explain the Phenomenon of Parity Inversion in ^{11}Be ?



Can we obtain an alternative description in terms of elementary modes of excitation?

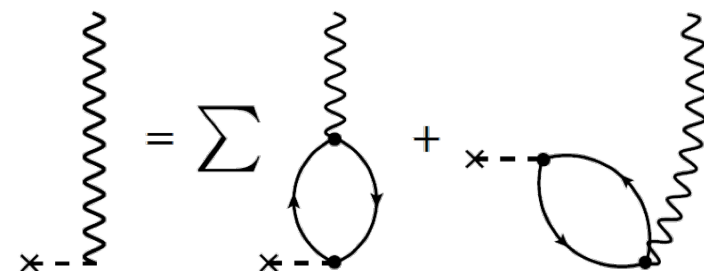
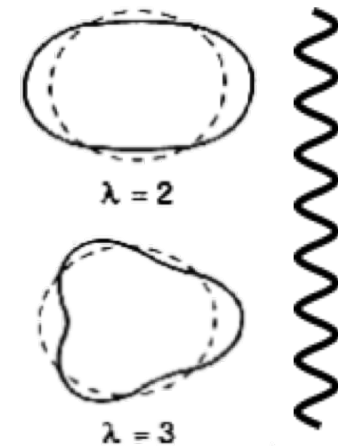
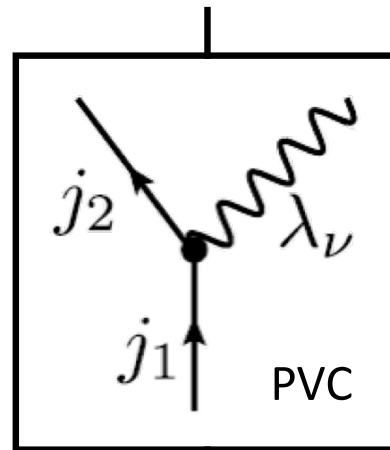
Independent Particles



Hartree-Fock mean Field

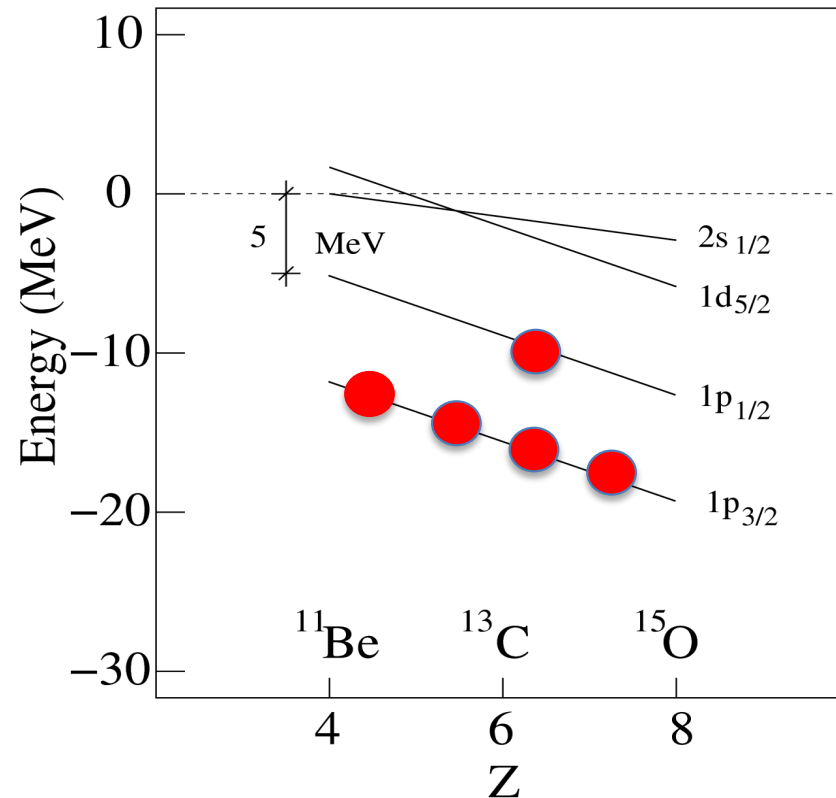
Collective Phonons

Particle-vibration coupling



Random Phase Approximation

Parity inversion in N=7 isotones is not reproduced by spherical mean field calculations



Typical spherical
mean-field results
with Skyrme forces
(Sagawa, Brown, Esbensen
PLB 309(93)1)

A possible explanation of parity inversion: dynamical coupling between the core and the loosely bound neutron

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Structure of Exotic Neutron-Rich Nuclei

Takaharu Otsuka, Nobuhisa Fukunishi, and Hiroyuki Sagawa

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(Received 13 November 1992)

A new framework, the variational shell model, is proposed to describe the structure of neutron-rich unstable nuclei. An application to ^{11}Be is presented. Contrary to the failure of the spherical Hartree-Fock model, the anomalous $\frac{1}{2}^+$ ground state and its neutron halo are reproduced with the Skyrme (SIII) interaction. This state is bound due to dynamical coupling between the core and the loosely bound neutron, which oscillates between the $2s_{1/2}$ and the $1d_{3/2}$ orbits.

The core: spherical or deformed?
Important role of fluctuations expected
We propose a dynamical description

N. Vinh Mau, Nucl. Phys. A 592 (1995) 43

G.F. Esbensen and H. Sagawa, Phys. Rev C 51 (1995) 1274

F.M. Nunes and I. Thompson, Nucl. Phys. A 703 (2002) 593

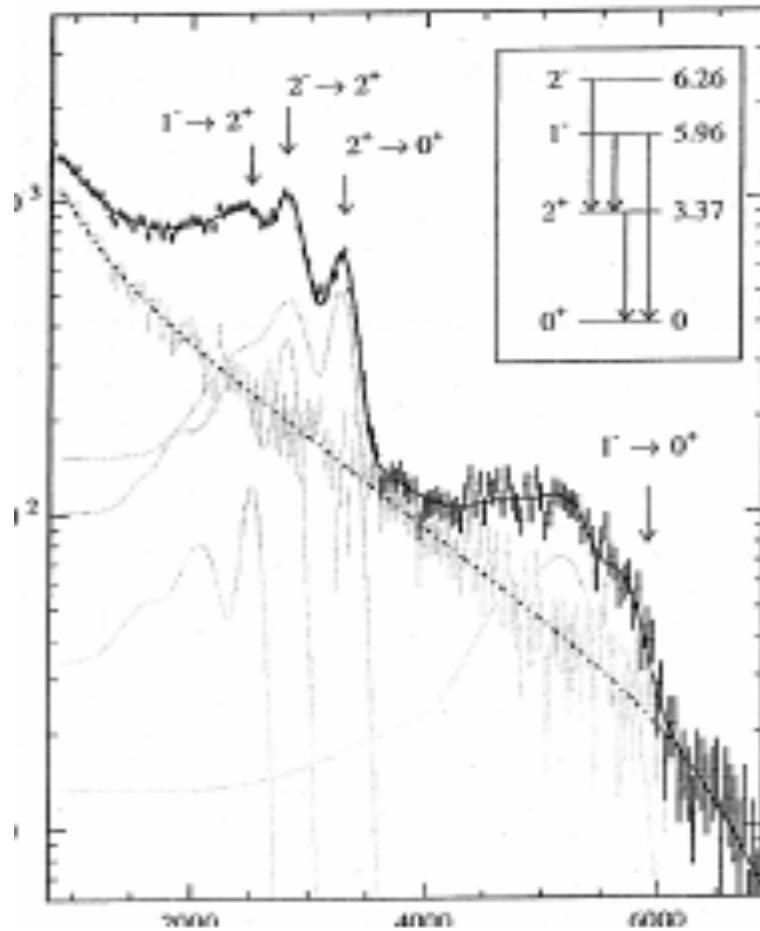
G. Blanchon et al., Phys Rev. C 82 (2010) 034313

Myo et al, PRC 86 (2012) 024318

I. Hamamoto and S. Shimoura, J. Phys. G 34 (2007) 2715

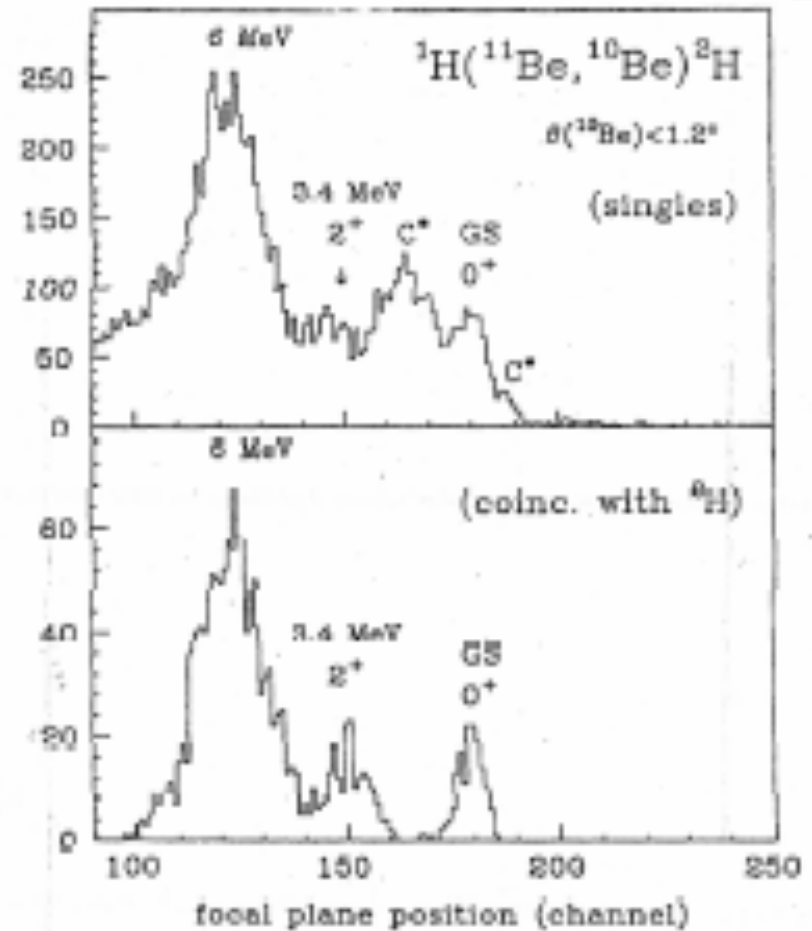
The admixture of $d_{5/2} \times 2^+$ configuration in the $1/2^+$ g.s. of ^{11}Be is about 15%

$9\text{Be}(^{11}\text{Be},^{10}\text{Be}+\gamma)\text{X}$



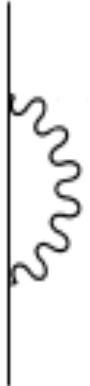
T. Aumann et al. PRL 84 (2000) 35

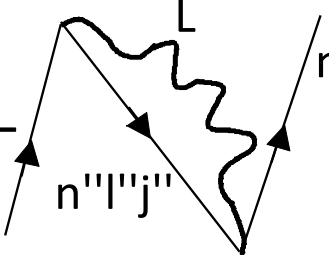
$p(^{11}\text{Be},^{10}\text{Be})d$

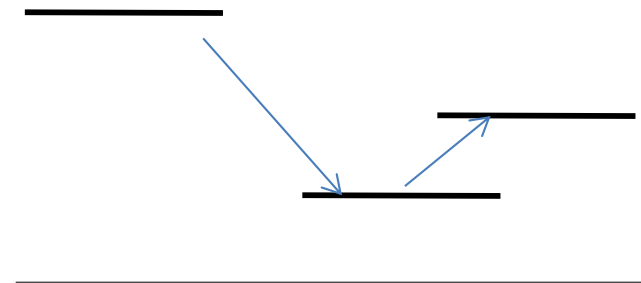


S. Fortier et al. Phys. Lett. B461 (1999)22
J.S. Winfield et al., Nucl.Phys. A683 (2001)48

Basic effect of particle-vibration coupling on the single-particle energies close to the Fermi energy

(A)  $L = \frac{h^2(j, j', L)}{e_j - (e_{j'} + \hbar\omega_\lambda)} < 0$

(B)  $= \frac{h^2(j, j', L)}{(e_j - e_{j''} + \hbar\omega_\lambda)} > 0$

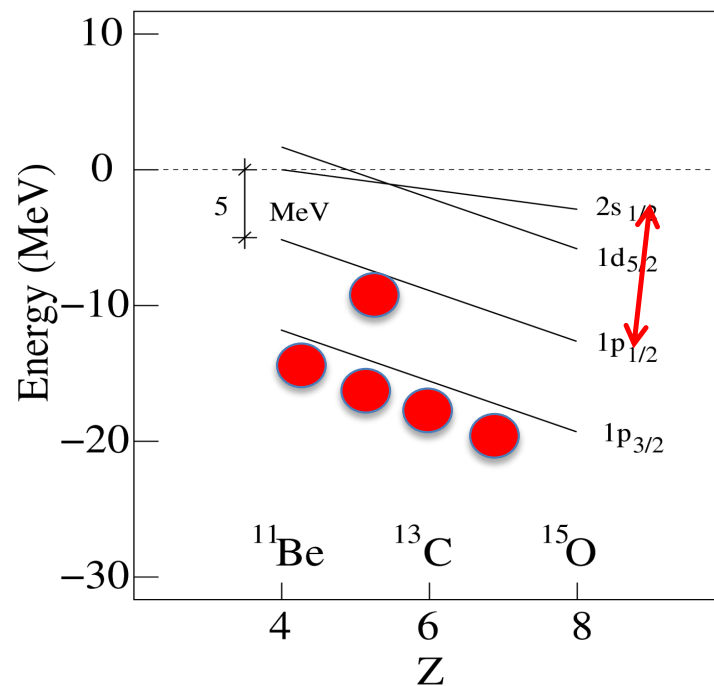
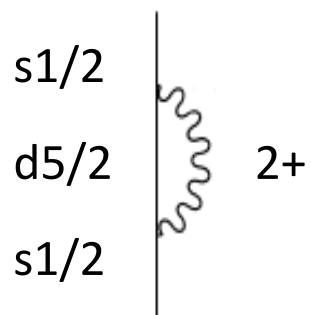


From B(EL) experimental value

$$h(a, b\lambda) = \frac{1}{\sqrt{4\pi}} \langle j_a \lambda | j_b \rangle \beta_\lambda \left\langle j_a \left| \frac{\partial U}{\partial r} \right| j_b \right\rangle$$

^{11}Be

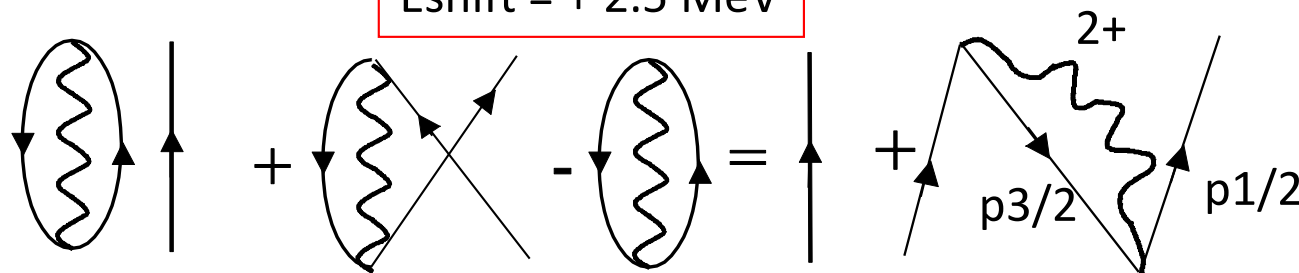
Eshift = - 2.5 MeV



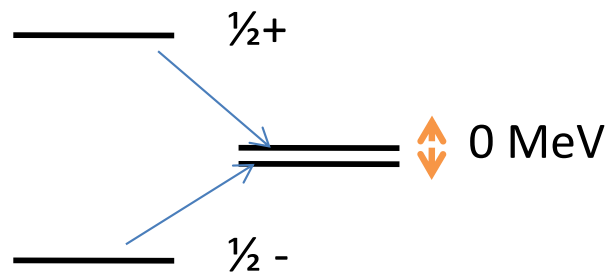
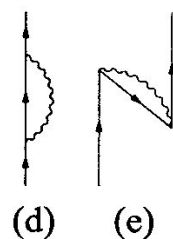
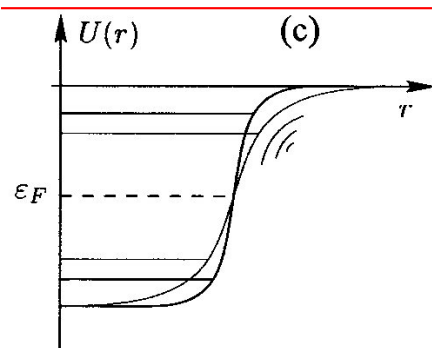
Self-energy

+

Eshift = + 2.5 MeV



Pauli blocking of
core ground state
correlations



Level inversion

$$B(E2) = 10.4 \pm 1.2 \text{ e}^2 \text{ fm}^4$$

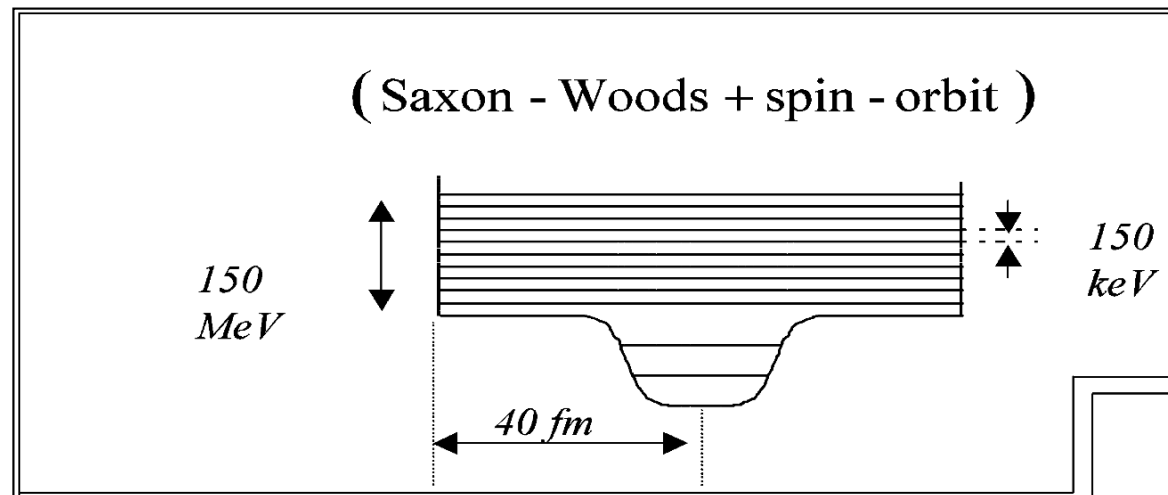
$$\beta_{\text{em}} = 1.12 \quad \beta_n \approx 0.9$$

Fermionic degrees of freedom:

- $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{5/2}$ Wood-Saxon levels in a box

Bosonic degrees of freedom:

- 2^+ , 3^- , pair vib. QRPA solutions tuned to reproduce available exp. data



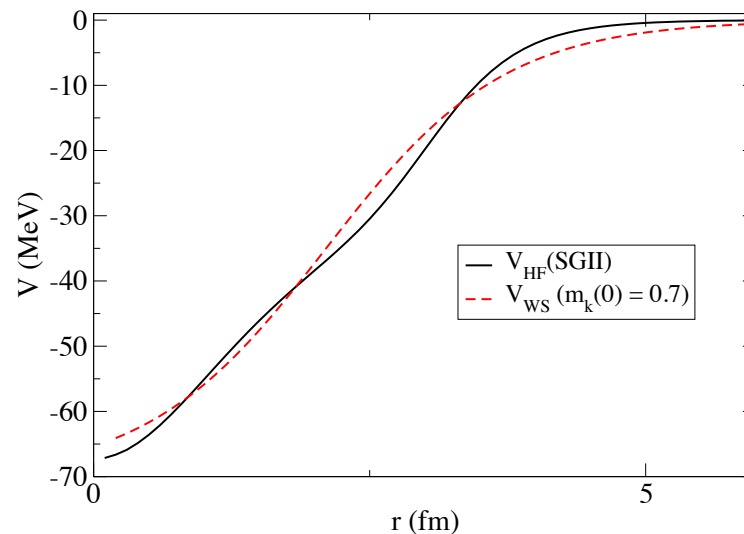
Parameter optimization

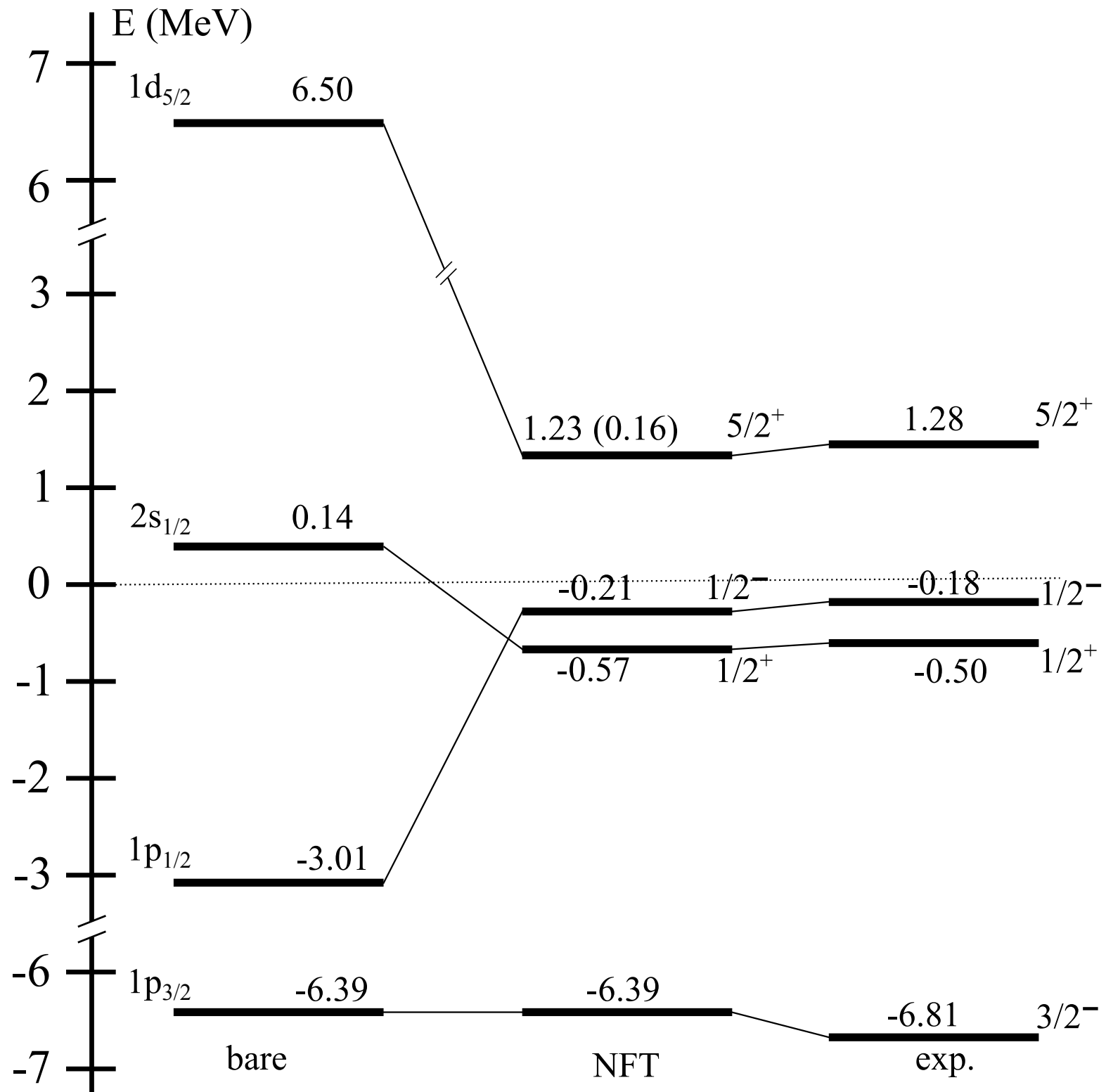
We perform the many-body calculation starting from a Woods-Saxon potential, with a spatially dependent effective mass, with

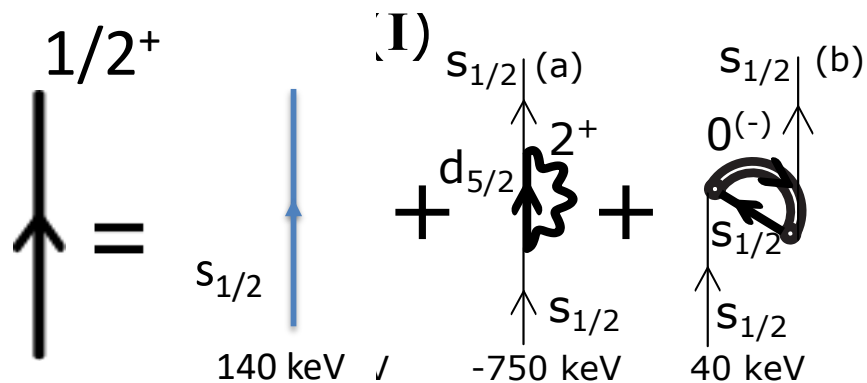
$$m_k(r=0) = 0.7 m, m_k(r \gg R) = \mu = 0.91 m$$

The following parameters are fitted to obtain the best agreement of the renormalized energies with the experimental $1/2^+, 1/2^-$ and $5/2^+$ states in ^{11}Be and $3/2^-$ in ^9Be :

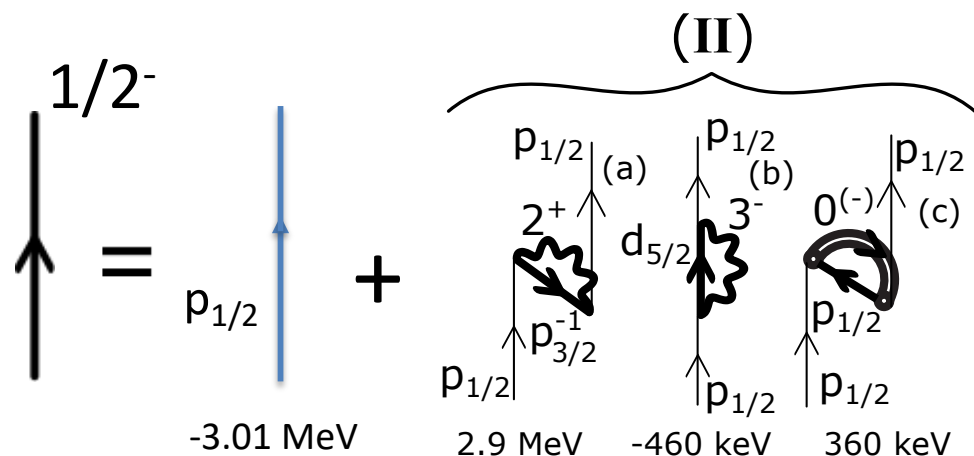
- Depth, diffuseness, radius, strength of spin-orbit coupling



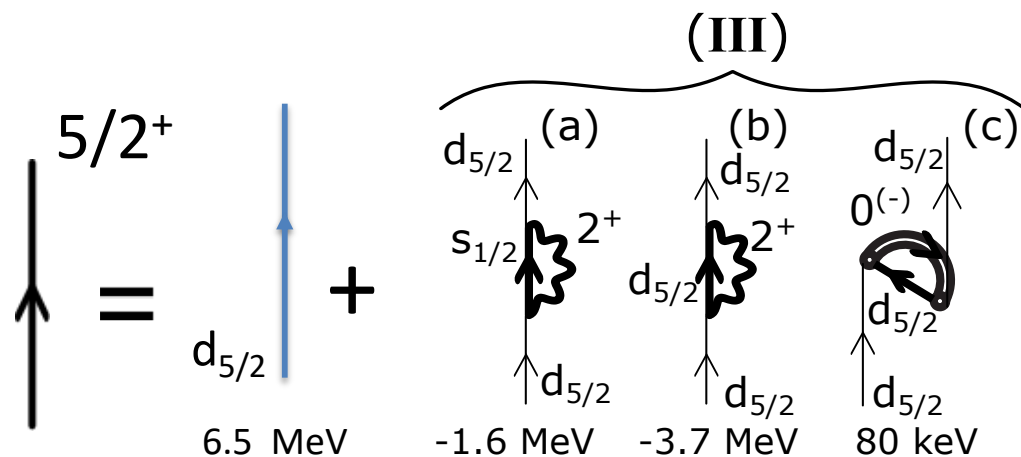




$$\sqrt{0.83}|s_{1/2} > + \sqrt{0.17}|(d_{5/2} \otimes 2^+)_{1/2^+} >$$

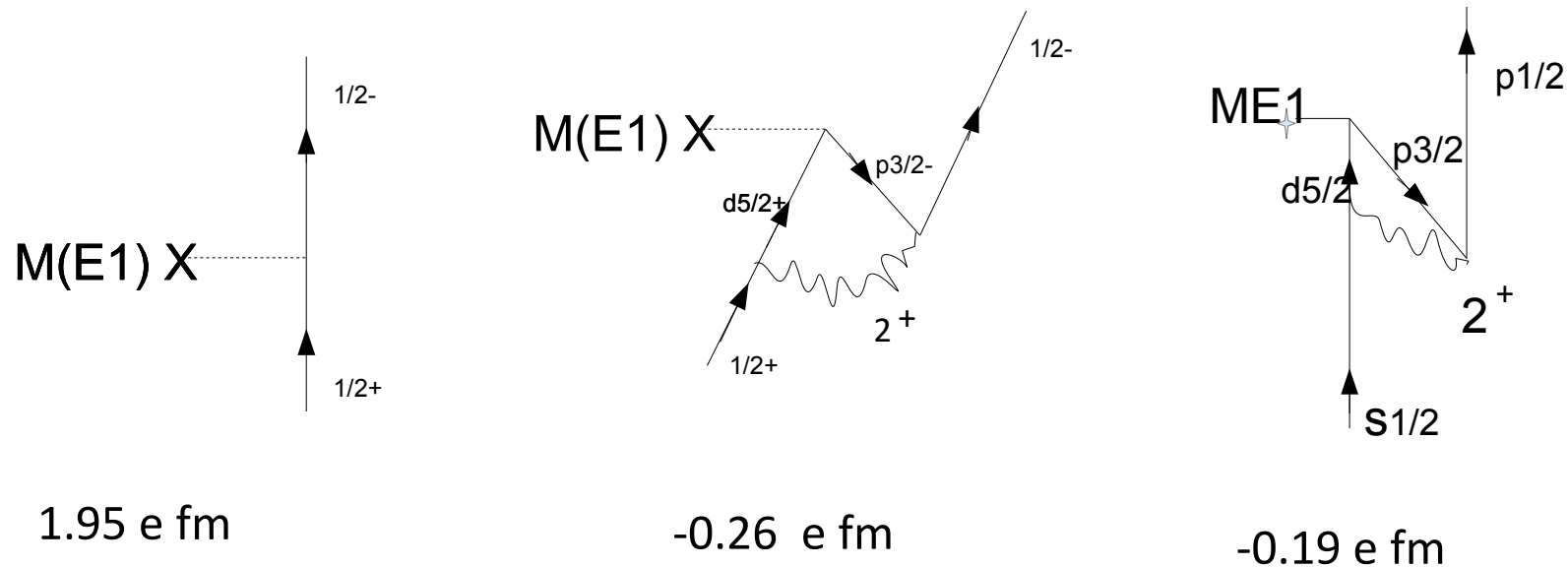


$$\sqrt{0.81}|p_{1/2} > + \sqrt{0.02}|(d_{5/2} \otimes 3^-)_{1/2^-} > \\ + \sqrt{0.15}|((p_{1/2}, 1p_{3/2}^{-1})_{2^+} \otimes 2^+) p_{1/2} >$$



$$\sqrt{0.34}|d_{5/2} > + \sqrt{0.32}|(s_{1/2} \otimes 2^+)_{5/2^+} > \\ + \sqrt{0.34}|(d_{5/2} \otimes 2^+)_{5/2^+} >$$

Strength of the dipole transition between $\frac{1}{2}^+$ and $\frac{1}{2}^-$ states



$$B(E1) \text{ (th.)} = 0.11 \text{ e}^2 \text{ fm}^2$$

$$B(E1) \text{ (exp.)} = 0.102 \pm 0.002 \text{ e}^2 \text{ fm}^2$$

This result is sensitive to the details of the mean field potential

Isotopic shift of the charge radius

$$(\langle r^2 \rangle_{10\text{Be}})^{1/2} = 2.361 \pm 0.017 \text{ fm} \quad (\langle r^2 \rangle_{11\text{Be}})^{1/2} = 2.466 \pm 0.015 \text{ fm}$$

Single-particle picture: $S=1$

$$(\langle r^2 \rangle)^{1/2}_{1s1/2} = 7.1 \text{ fm}$$

Many-body picture: $S=0.83$

$$(\langle r^2 \rangle)^{1/2}_{d5/2,\text{coll}} = 3.0 \text{ fm}$$


$$\begin{aligned} \langle r^2 \rangle_{11\text{Be}} = & \left(\langle r^2 \rangle_{10\text{Be}} + \left(\frac{\langle r^2 \rangle_{1s1/2}^{1/2}}{11} \right)^2 \right) \times S^2 + (1 - S^2) \times \left(\langle r^2 \rangle_{10\text{Be}} \left(1 + \frac{2}{4\pi} \beta_\pi^2 \right) + \left(\frac{\langle r^2 \rangle_{d5/2\text{coll}}^{1/2}}{11} \right)^2 \right) = \\ & \left(\langle r^2 \rangle_{10\text{Be}} + \left(\frac{\langle r^2 \rangle_{1s1/2}^{1/2}}{11} \right)^2 \right) \times S^2 + (1 - S^2) \times \left(\left(\frac{\langle r^2 \rangle_{d5/2\text{coll}}^{1/2}}{11} \right)^2 + \langle r^2 \rangle_{10\text{Be}} \frac{2}{4\pi} \beta_\pi^2 \right) \end{aligned}$$

$$\Delta \langle r^2 \rangle_{11\text{Be}}^{1/2} (\text{th.}) = 0.12 \text{ fm} / 0.27 \text{ fm}$$

$$\Delta \langle r^2 \rangle_{11\text{Be}}^{1/2} (\text{exp.}) = 0.11 \text{ fm}$$


Matrix elements due to GSC Pauli rearrangement

The contribution of a given p-h configuration to the GS Correlation Energy is (B&MII)



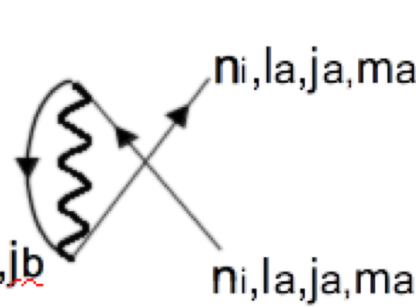
$$\delta E = \frac{(-h_{ai,bk,\lambda} \sqrt{(2j_a+1)})^2}{0 - (E_{ai} + E_{bk} + \hbar\omega_\lambda)} < 0$$

The presence of a new neutron (scattering- or bound-like) inhibits some of these correlations, producing an energy modification of the core state...



$$-\frac{\delta E}{2j_a+1} > 0$$

This is the meaning/value of the NFT self energy diagram (B&MII, eq.6.225)



$$(-1) \frac{(-h_{ai,bk,\lambda} \sqrt{2j_a+1})^2 \langle ((j_{a1}, j_{a2}) J=0, j_{a3}; j_a | (j_{a1}, j_{a3}) J=0, j_{a2}; j_a) \rangle}{E_{ai} - (2E_{ai} + E_{bk} + \hbar\omega_\lambda)}$$

$$= \frac{(h_{ai,bk,\lambda})^2}{(E_{ai} + E_{bk} + \hbar\omega_\lambda)}$$

One more auxiliary GSCPR channel.

$$\begin{aligned} u_a^x(r) &= \sum_i x_{ai} R_{ai}^{WS}(r); e_{ai} > e_F \\ u_b^C(r) &= \sum_i C_{bi} R_{bi}^{WS}(r); e_{bi} > e_F \\ v_c^D(r) &= \sum_i D_{ci} R_{ci}^{WS}(r); e_{ci} < e_F \\ v_a^y(r) &= \sum_i y_{ai} R_{ai}^{WS}(r); e_{ai} < e_F \end{aligned}$$

$$\Psi_{jama} = [\psi_{jama}^x + [\psi_{jb}^C \cdot \Gamma_{\lambda}^+]_{jama}] \Phi_{GS}$$

but if

$$\Psi_{GS} = [1 + \epsilon [\psi_{ja,occ}^{-1} [\psi_{jb}^x \cdot \Gamma_{\lambda}^+]_{ja}]]_{J=0} \Phi_{GS}^{HF}$$

a new term can / must be added

$$\Psi_a = [\psi_a^x + [\psi_b^C \cdot \Gamma_{\lambda}^+]_{ja} - \psi_a^y - [\psi_c^D \cdot \Gamma_{\lambda}]_{ja} + \dots] \Phi_{GS}$$

with the hole annihilator

$$\psi_a^y = (v_a^y(r)/r) \Theta_{ja},$$

$$v_a^y(r) = \sum_i y_{ai} R_{ai}^{WS}(r); e_{ai} < e_F$$

p-h + phonon
"virtual" excitations
with respect to the HF GS.

Creation of a neutron in an
"occupied" level ja level
may/will give a non rule
contribution

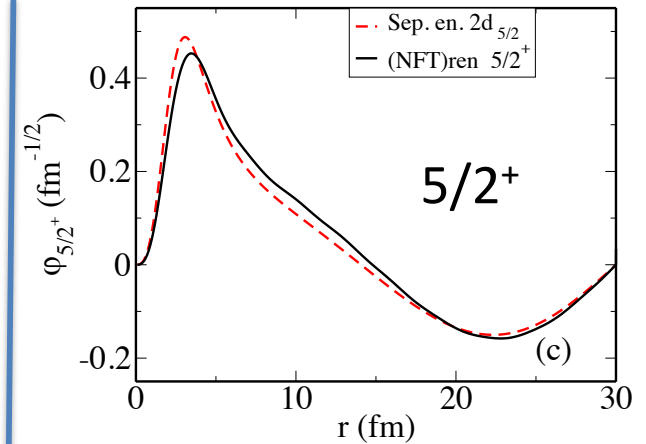
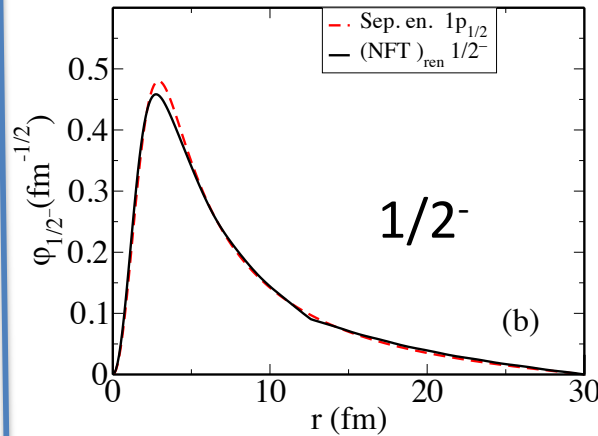
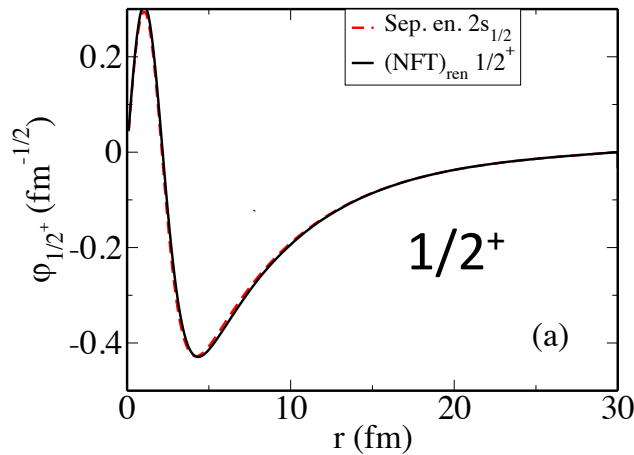
GSC Pauli rearr.:
An auxiliary
Coupled Channel

$$\begin{pmatrix} H_p - e_F & \Xi_{a,b\lambda} f(r) & 0 & \Xi_{a,c\lambda} f(r) \\ \Xi_{a,b\lambda} f(r) & H_p - e_F + \hbar\omega & \Xi_{a,b\lambda} f(r) & 0 \\ 0 & \Xi_{a,b\lambda} f(r) & (H_p - e_F) & -\Xi_{a,c\lambda} f(r) \\ \Xi_{a,c\lambda} f(r) & 0 & -\Xi_{a,c\lambda} f(r) & (H_p - e_F) - \hbar\omega \end{pmatrix} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_a^y \\ -v_c^D \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_a^y \\ -v_c^D \end{pmatrix}$$

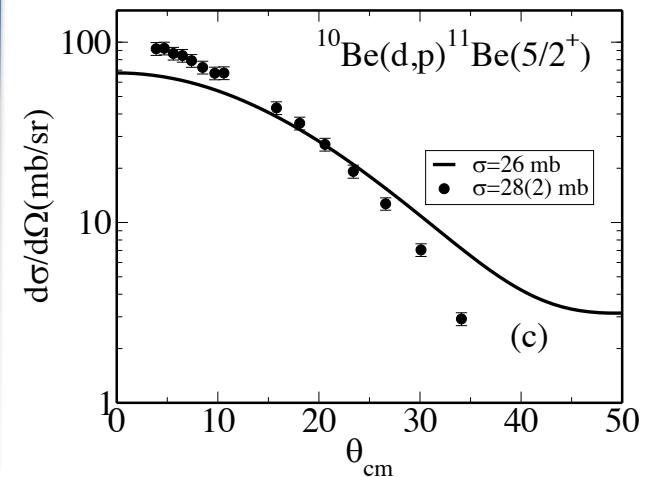
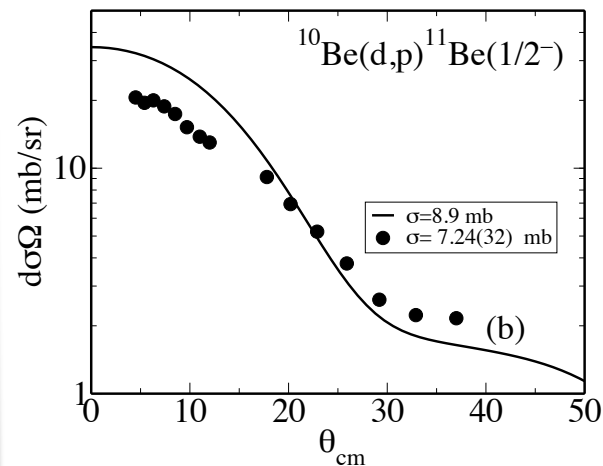
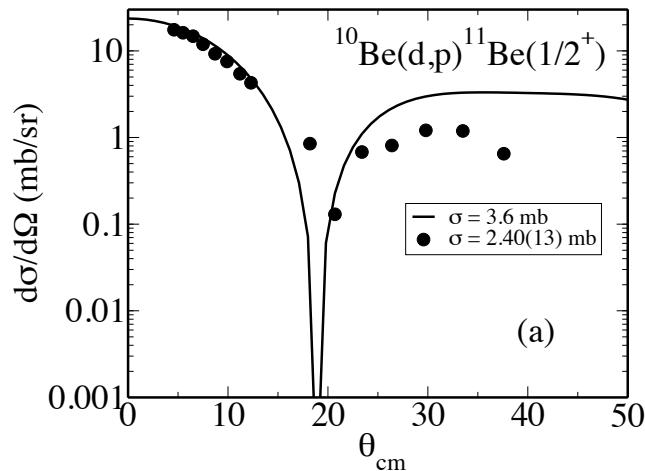
$^{10}\text{Be}(d,p)^{11}\text{Be}$ at $E_d = 21.4$ MeV

Test of the single-particle component of the many-body wavefunction

Form factors



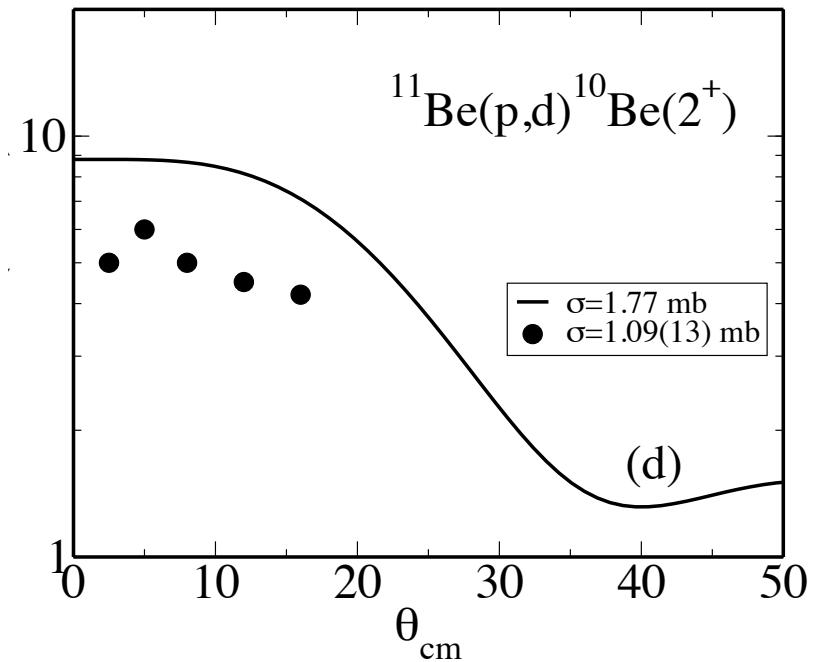
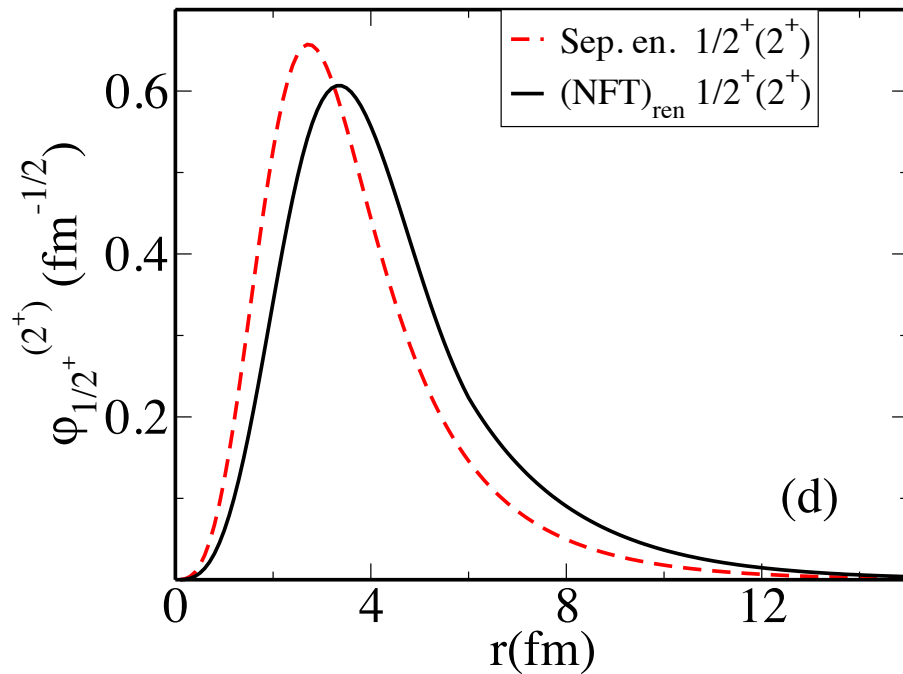
Cross sections



$^{11}\text{Be}(1/2^+)(p,d)^{10}\text{Be}(2^+)$

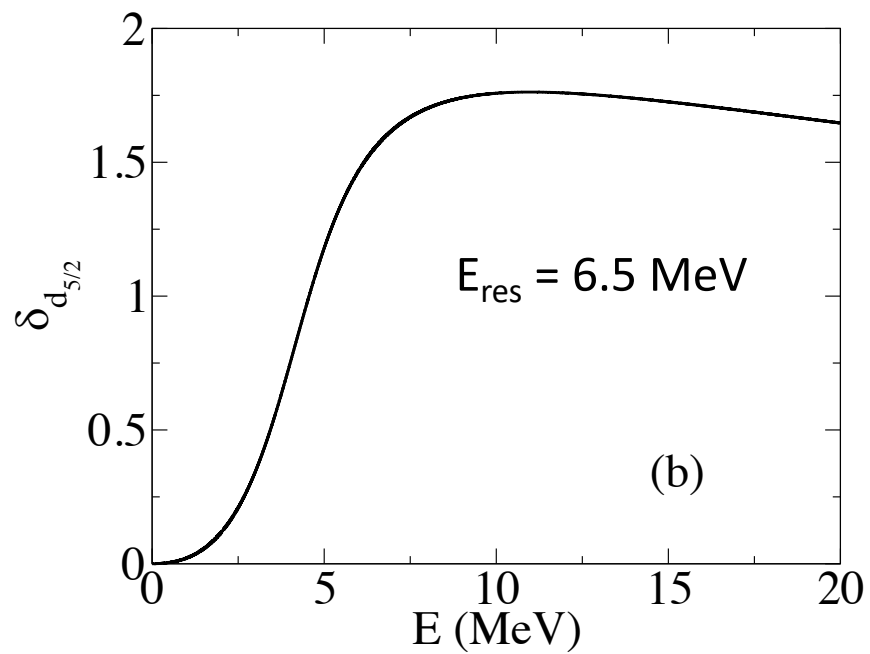
Test of the collective component $R_{d5/2}^C$ of the many-body wavefunction
(but we should calculate the optical potential microscopically!)

$$(\psi_b^C \otimes \Gamma_\lambda^+)_{j_a} = (R_b^C(r)/r)(\Theta_{j_b} \otimes \Gamma_\lambda^+)_{j_a}$$

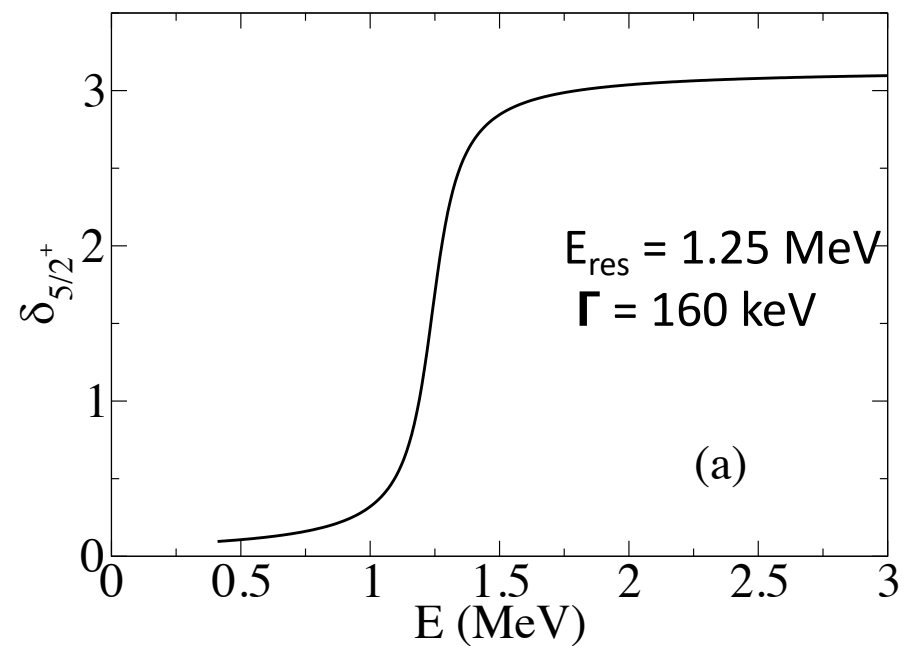


J.S. Winfield et al, Nucl. Phys. A 683, 48 (2001)

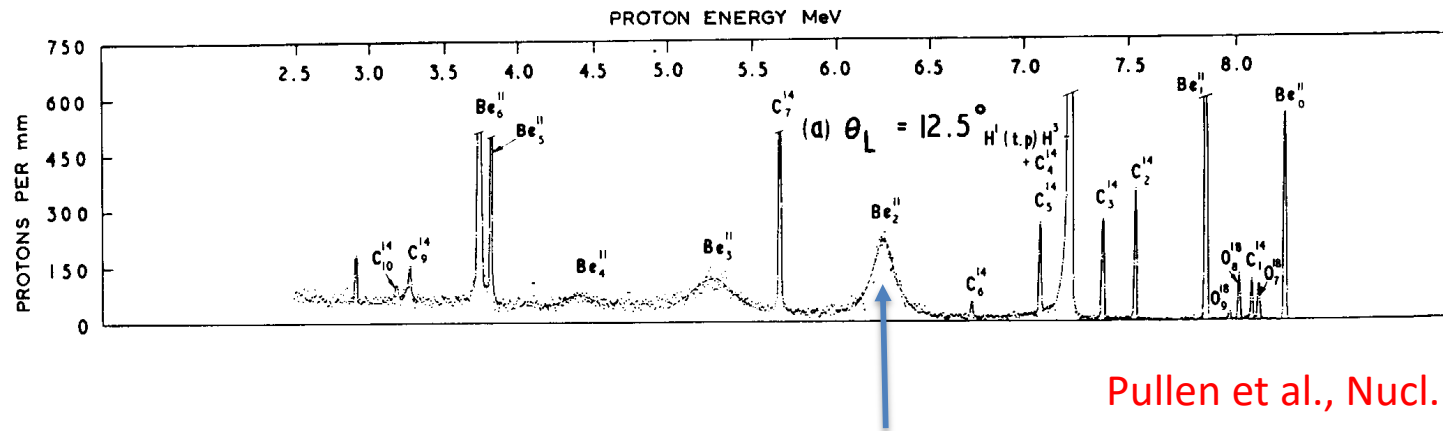
d5/2 phase shift in the bare potential



Renormalized 5/2+ phase shift

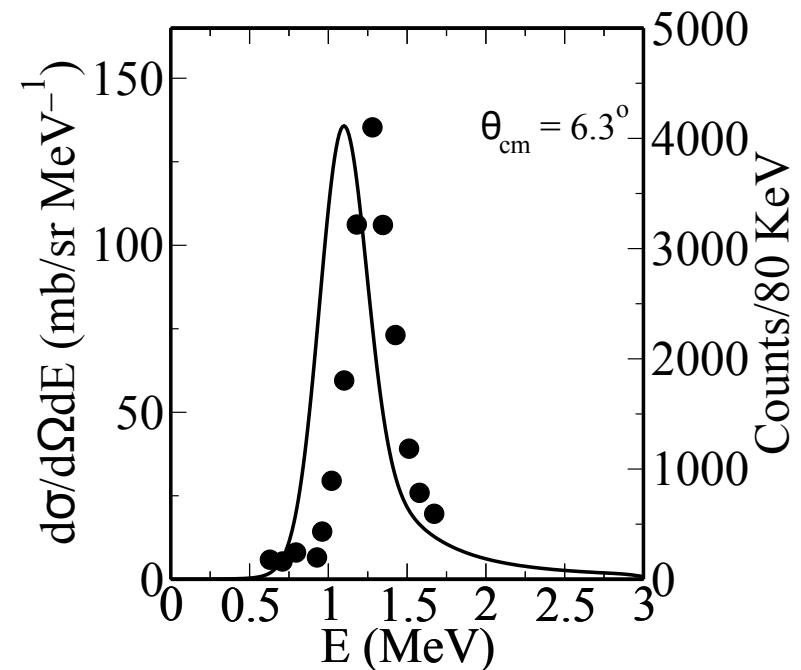
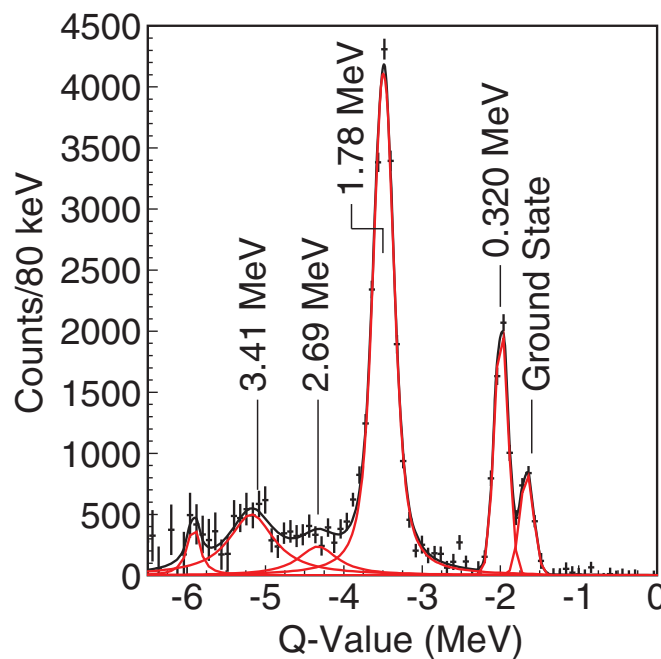


The usually quoted value of width of the $5/2^+$ resonance (100 keV) is derived from ${}^9\text{Be}(t,p){}^{11}\text{Be}$ spectra



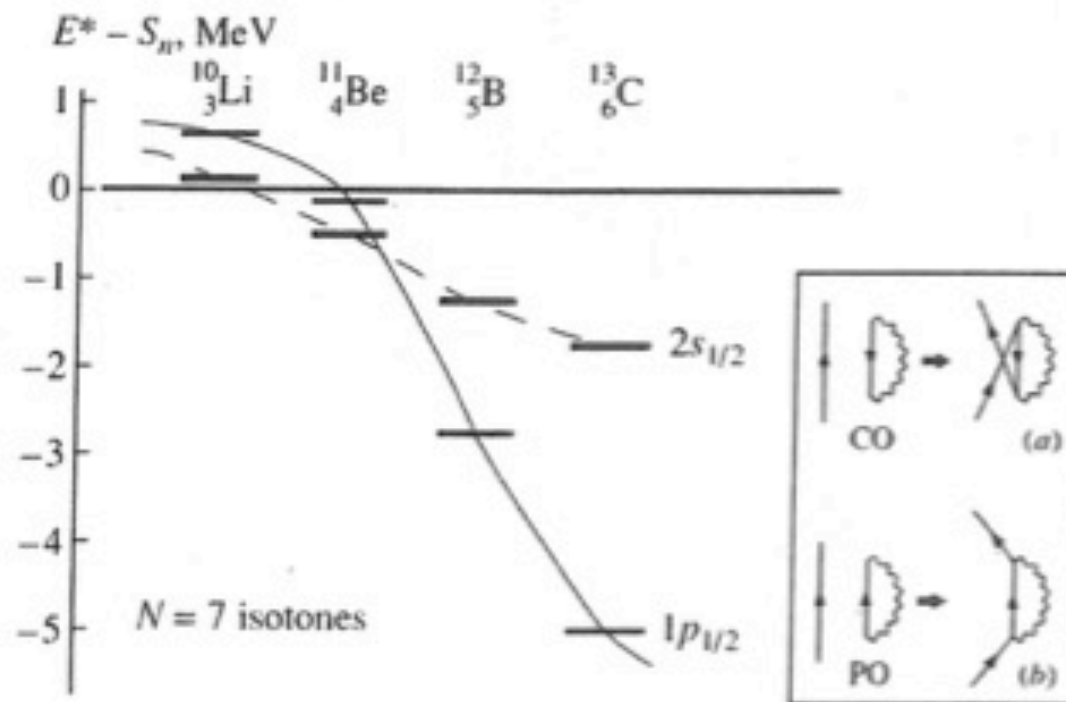
Pullen et al., Nucl. Phys. 36 (1962)1

The width from ${}^{10}\text{Be}(d,p){}^{11}\text{Be}(5/2^+)$ spectra is much larger and is well reproduced by theory

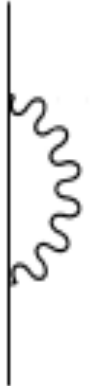


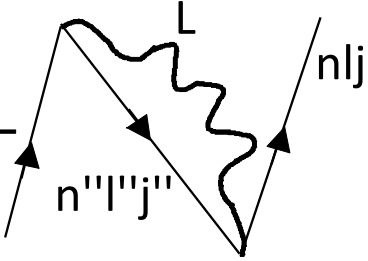
It is possible to obtain a quantitative description of the structure and of the reactions of ^{11}Be , based on the dynamical coupling of particles and vibrations, taking properly into account ground state correlations

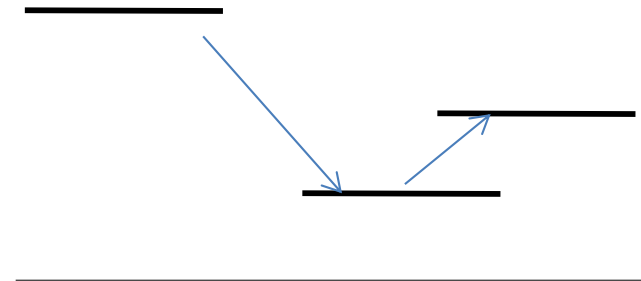
Extend the calculations and check theory in neighbouring nuclei $^{11}\text{N}, (^{10}\text{Li}, ^{12}\text{B}, ^{13}\text{C})$



Basic effect of particle-vibration coupling on the single-particle energies close to the Fermi energy

(A) 
$$L = \frac{h^2(j,j',L)}{e_j - (e_{j'} + \hbar\omega_\lambda)} < 0$$

(B) 
$$+ \frac{h^2(j,j',L)}{(e_j - e_{j''} + \hbar\omega_\lambda)} > 0$$



This is a UNIVERSAL RESULT: Green's function, Equations of Motion, in general any many-body theory based on single-particle picture

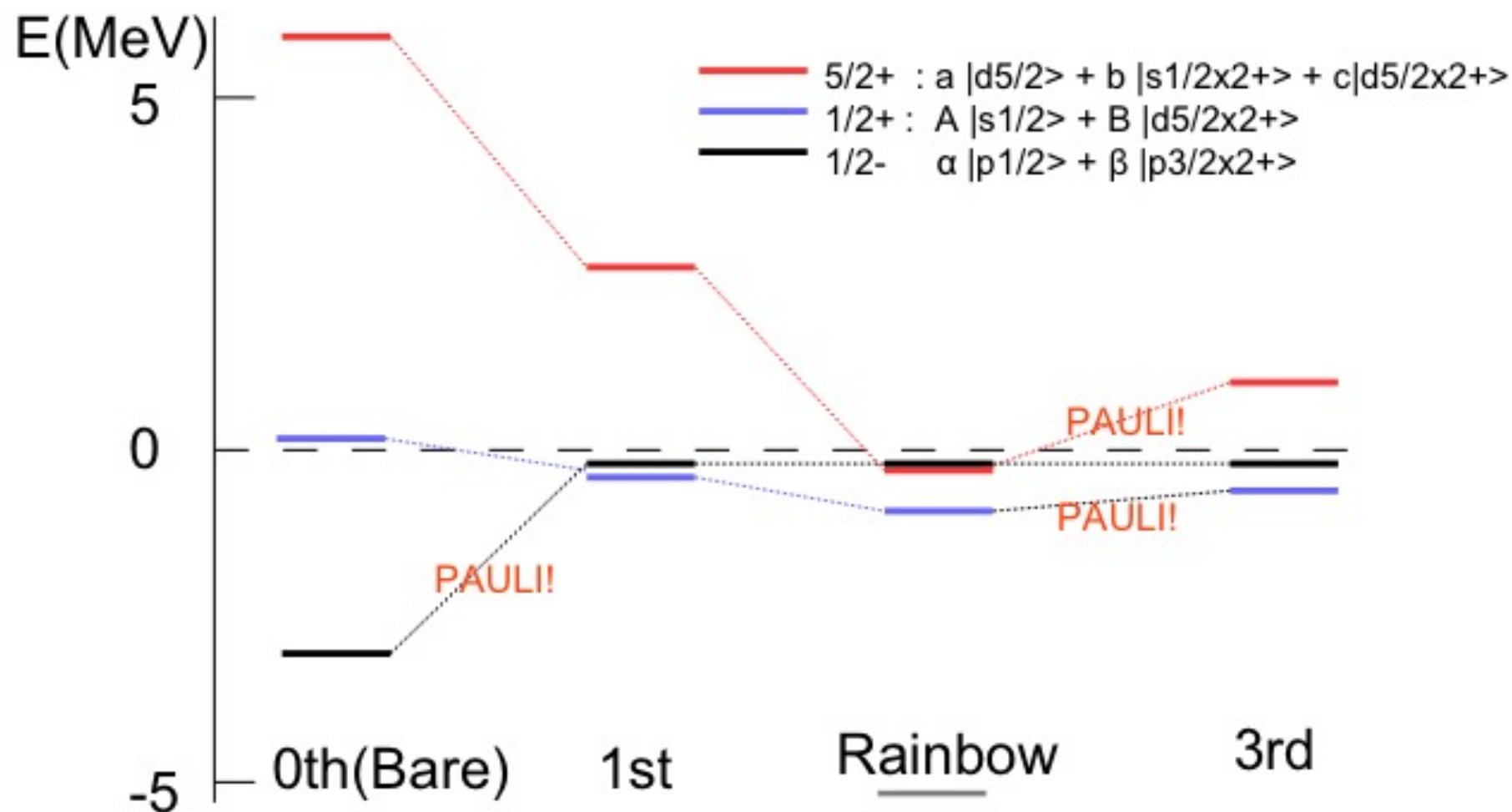
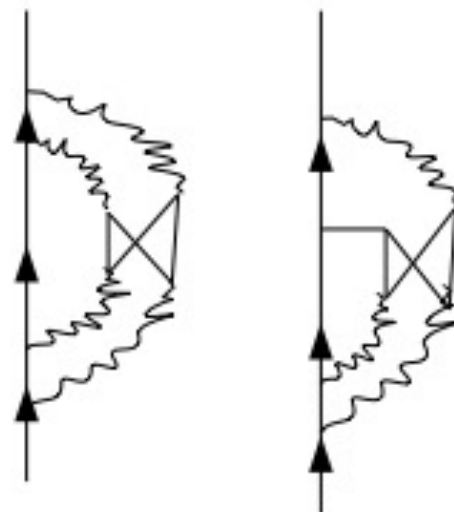
$$V_0 = 70 \text{ MeV}$$

$$a = 0.81 \text{ fm}$$

$$R = 2.1 \text{ fm}$$


Two Phonon Anharmonicities

Butterfly Diagrams

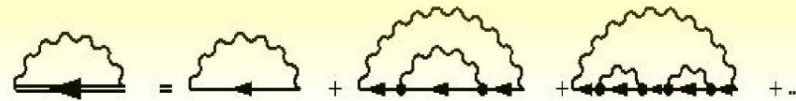


Microscopic description of superfluid nuclei beyond mean field: iterating the PVC with Nambu-Gor'kov formalism

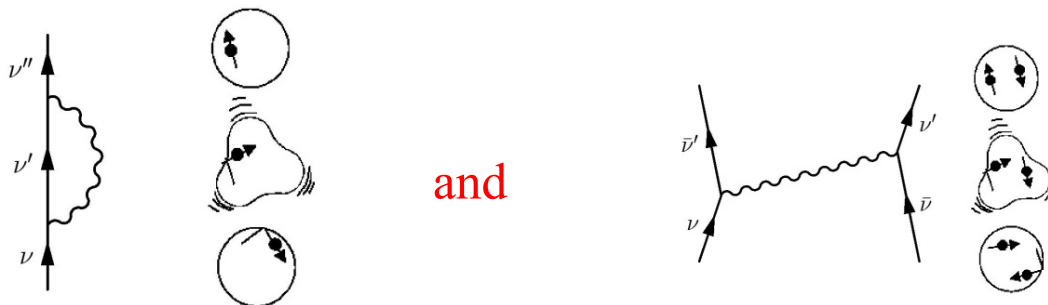
by extending the Dyson equation...

$$G_{\mu}^{-1} = (G_{\mu}^o)^{-1} - \Sigma_{\mu}(\omega)$$


$$\Sigma_{\mu}(\omega) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \sum_{\mu'} \frac{1}{\hbar} G_{\mu'}(\omega') \sum_{\alpha} \frac{1}{\hbar} D_{\alpha}^o(\omega - \omega') * V_{\mu\mu',\alpha}^2$$



... to the case of superfluid nuclei (Nambu-Gor'kov), it is possible to consider both:



J. Terasaki et al., Nucl.Phys. **A697**(2002)126;
F. Barranco et al, EPJ **A21** (2004) 57
A. Idini et al. PRC **85** (2012) 014
cf. V. Soma', C. Barbieri, T. Duguet,
PRC **84** (2011) 064317 ;PRC**87** (2013) 011303

Based on this approach, we could calculate several nuclear structure observables in ^{120}Sn with a 10% error

Observables	Opt. levels
Δ (keV)	50 (3.5 %)
E_{qp} (keV)	45 (4.5 %)
Mult. splitt. (keV)	59 (8.4 %)
$d_{5/2}$ (centr.) (keV)	40 (4%)
$d_{5/2}$ (width) (keV)	8 (1%)
$B(E2)/B_{sp}$	1.43 (14%)
$\sigma_{2n}(p, t)$ (mb)	40 (2%)