Description of spontaneous fission & its hindrance: odd-N or odd-Z nuclei & isomers

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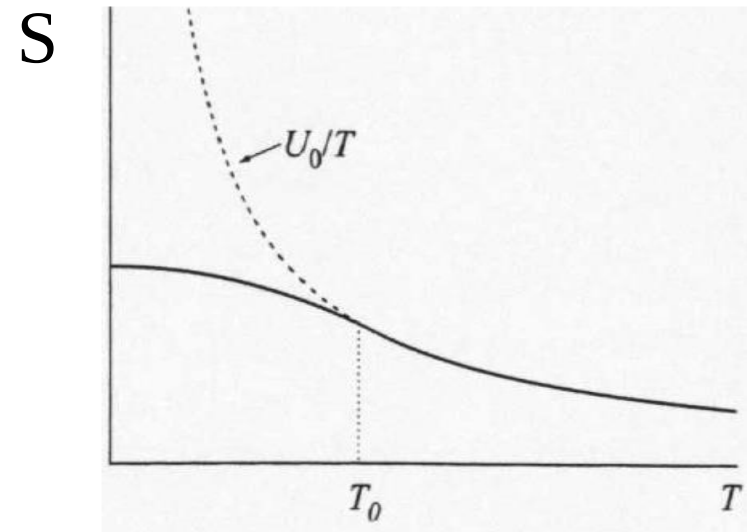
- **Special interest (beyond obvious): Possible long-living isomers in e-e SHN and ground states & isomers in odd & odd-odd SHN**
- **Data on isomer& odd-A fission half-lives & ways to understand them**
- **A difficulty in calculating action for odd-A nuclei or isomers – a breakdown of the adiabatic approximation**
- **A possible solution - instanton-motivated selfconsistent or non-selfconsistent approach**

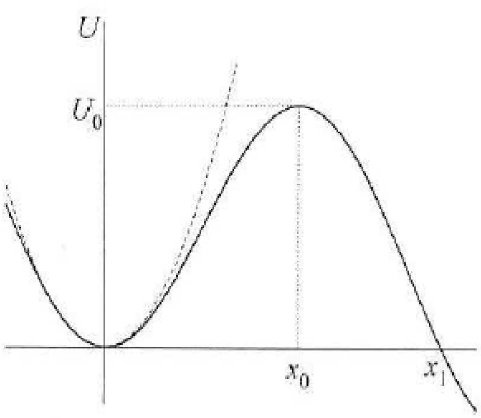
$\Gamma = A(T)\exp[-S(T)]$

$$S(T) = \frac{1}{\hbar}\int_0^{\hbar/T} d\tau \left(\frac{1}{2}\dot{x}_\tau^2 + U(x)\right)$$

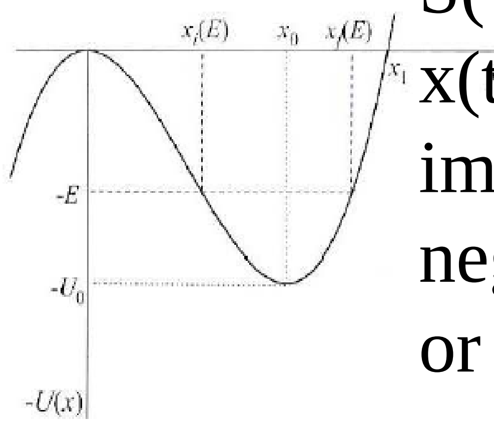


Dynamics in real and imaginary time

$L = T - V$



$L = -(T + V)$



S(T) – action;
x(tau)- a motion in the imaginary time – with a negative kinetic energy or in the inverted potential

For a nucleus, enters the position-dependent mass.

The action corresponding to a trajectory L (tunneling path) between two points $\boldsymbol{A}$ and $\boldsymbol{B}$ in a q-space is

$$S(\boldsymbol{A}, \boldsymbol{B}, L, E_0) = \int_L \sqrt{2B_L(\boldsymbol{q}(s))[V(\boldsymbol{q}(s)) - E_0]}\, ds$$

where the effective mass parameter B_L associated with the trajectory L is defined as:

$$B_L = \sum_{i,j} B_{ij} \frac{dq_i}{ds} \frac{dq_j}{ds}$$

To calculate the fission rate Γ we need to find the trajectory that minimize the action S:

$$\Gamma \propto exp\left[-\frac{2}{\hbar} S_{min}\right] \qquad T_{1/2} = \frac{ln2}{\Gamma}$$

$$HF(Z,N) = T_{SF,exp}(Z,N)/T_{ee}(Z,N),$$

$$T_{ee}(Z,N) = (T_{SF}(Z,N-1) \times T_{SF}(Z,N+1))^{1/2}$$

$$T_{ee}(Z,N) = (T_{SF}(Z-1,N) \times T_{SF}(Z+1,N))^{1/2}$$

Odd nuclei- a hindrance of fission

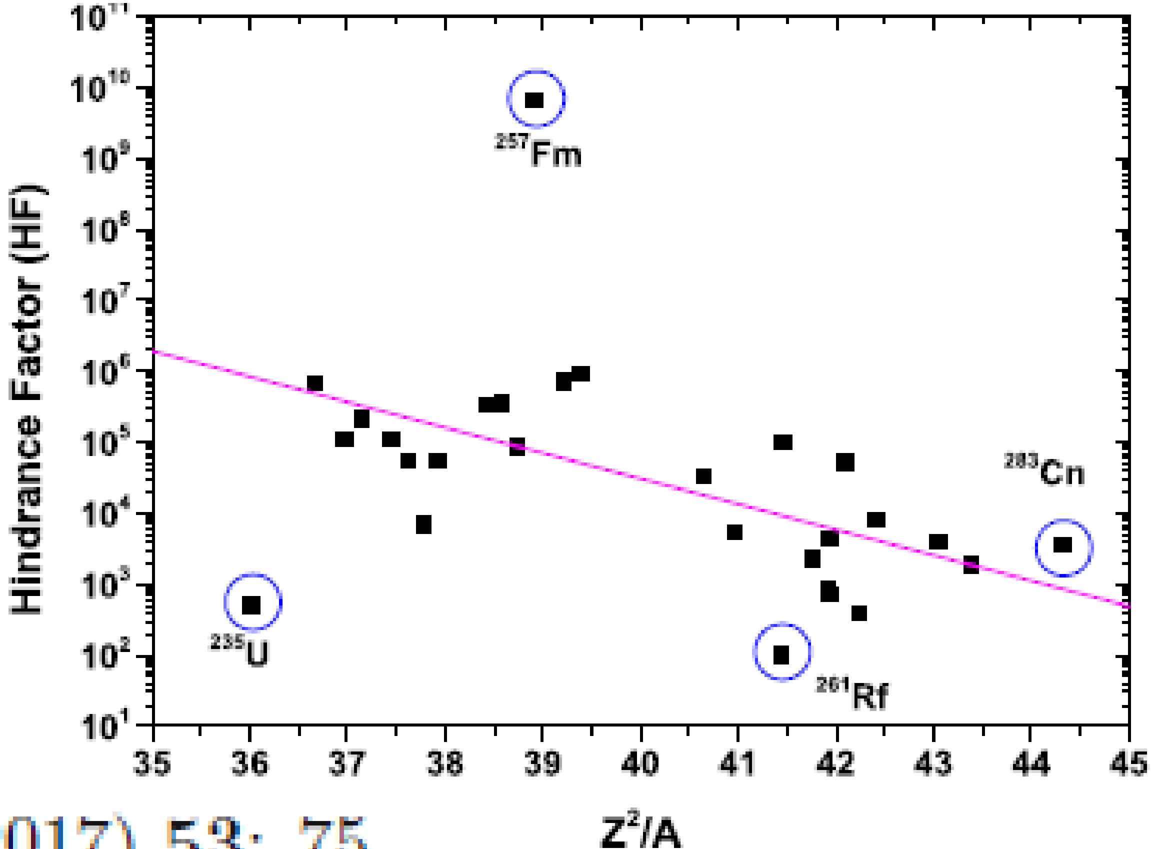


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Fig. 19. Spontaneous fission hindrance as a function of the fissility of the fissioning nucleus, expressed by Z^2/A. The line is to guide the eyes.

Invoked reasons for Tsf increase:

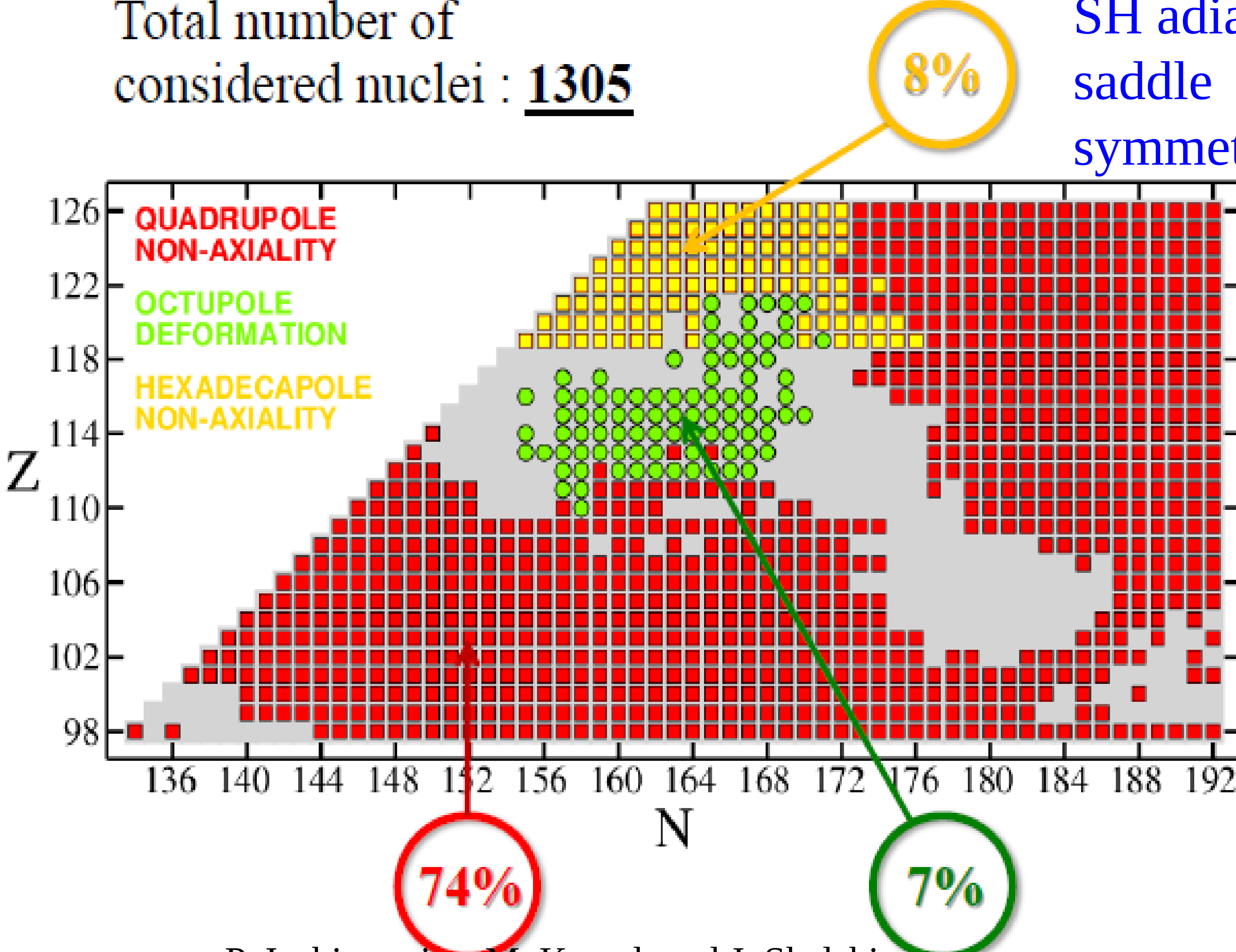
-for odd-Z or odd-N (vs. even), a smaller pairing gap causes an increase in the fission barrier and in the mass parameter (as given by a cranking expression);

- blocking a specific configuration additionally rises the barrier (provided it is conserved in fission) – specialization.

In calculations: the effect of keeping high-K number may be huge; if one does not suppress it, it seems the resulting half-lives in odd-A nuclei must come out too large.

Total number of considered nuclei : **1305**

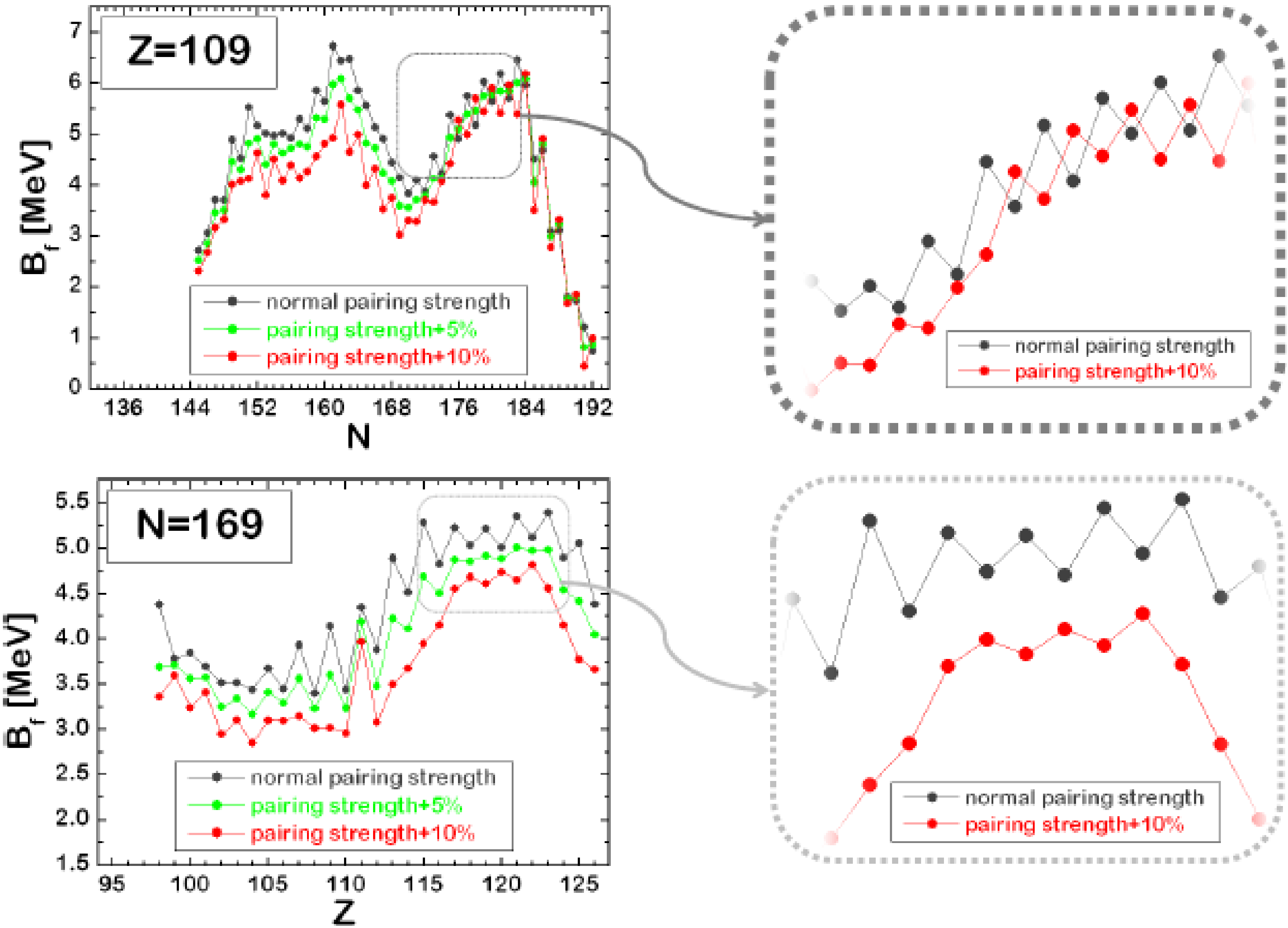
SH adiabatic saddle symmetry



P. Jachimowicz, M. Kowal, and J. Skalski

Phys. Rev. C 95, 014303 (2017)

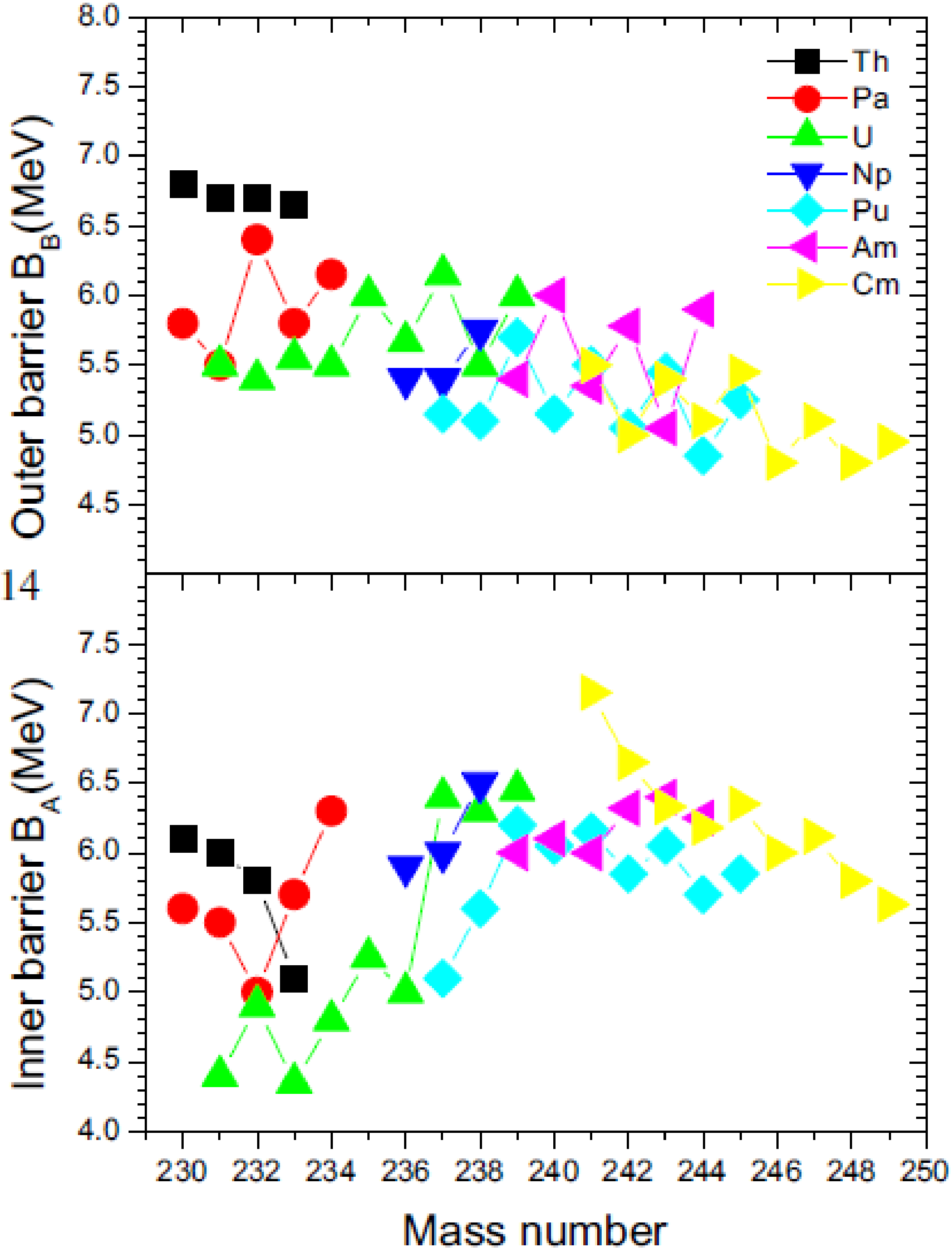
ODD-EVEN STAGGERING IN B_f vs. PAIRING STRENGTHS



„Experimental” odd-even barrier staggering in actinides

R. Capote *et al.*

Nuclear Data Sheets 110 (2009) 3107–3214



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Larger Omega does not give larger HF

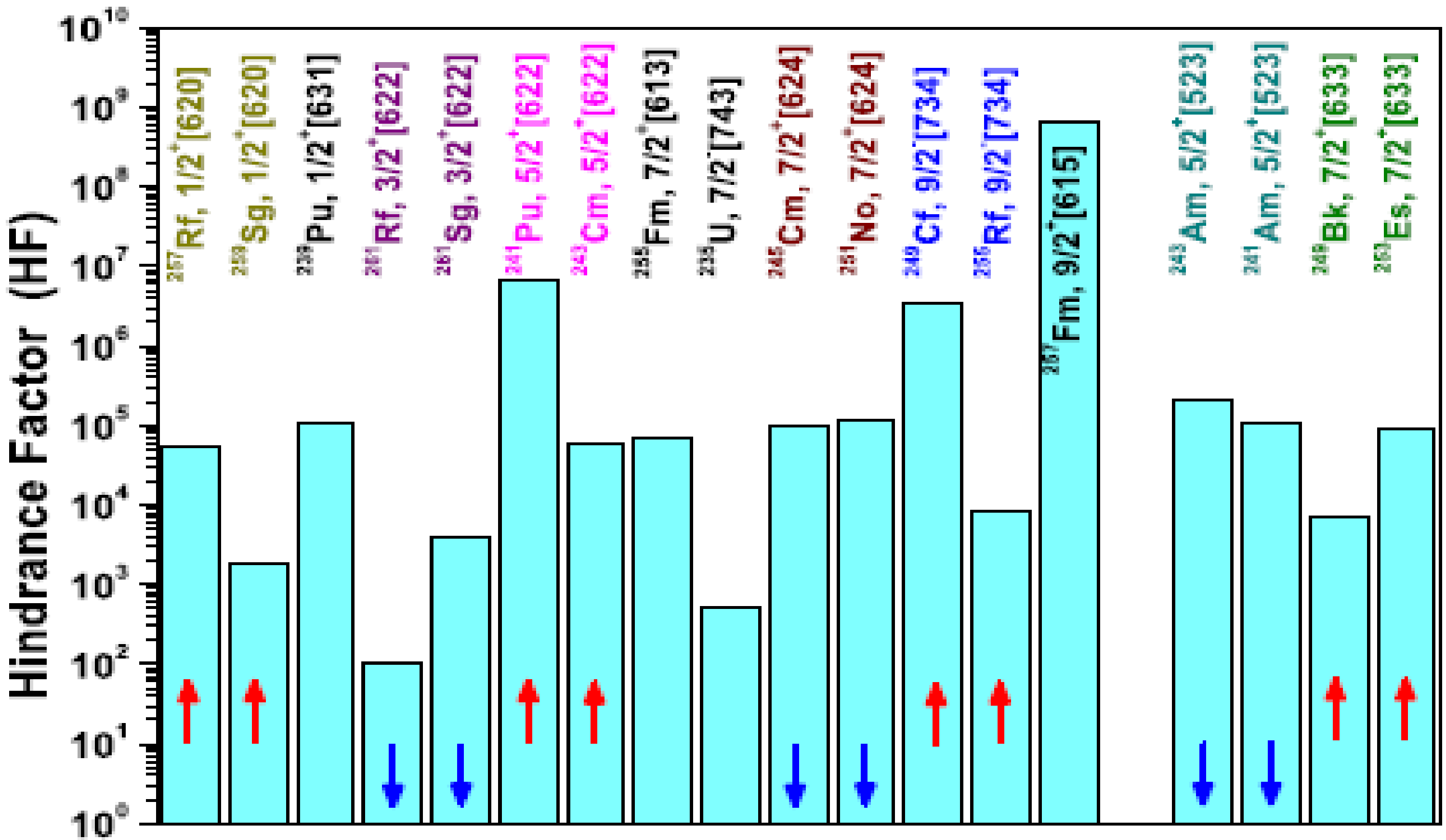
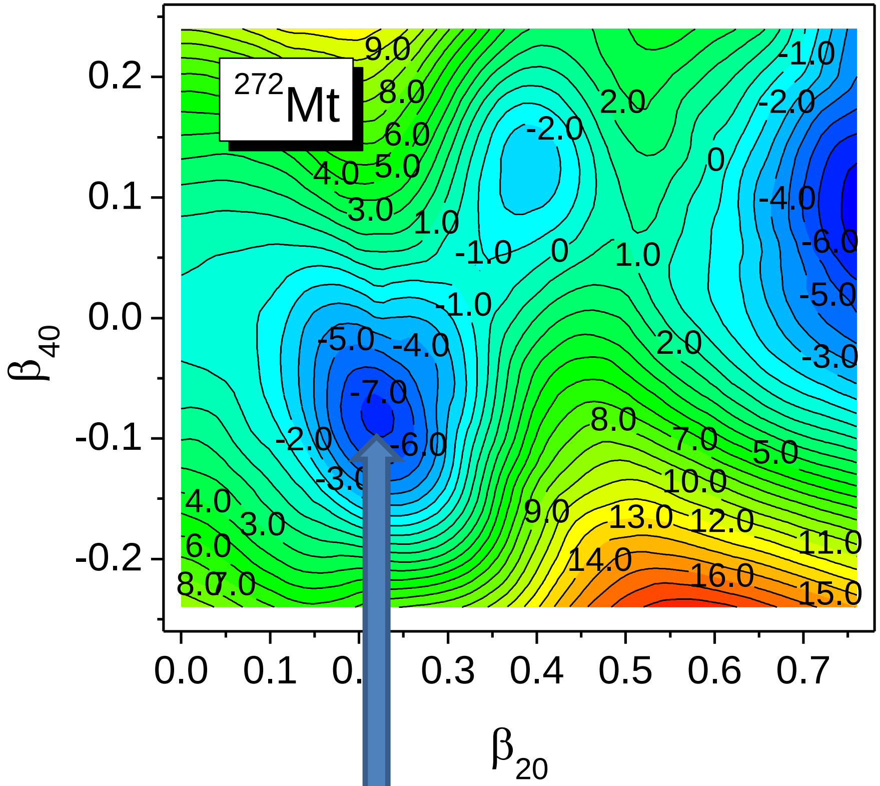
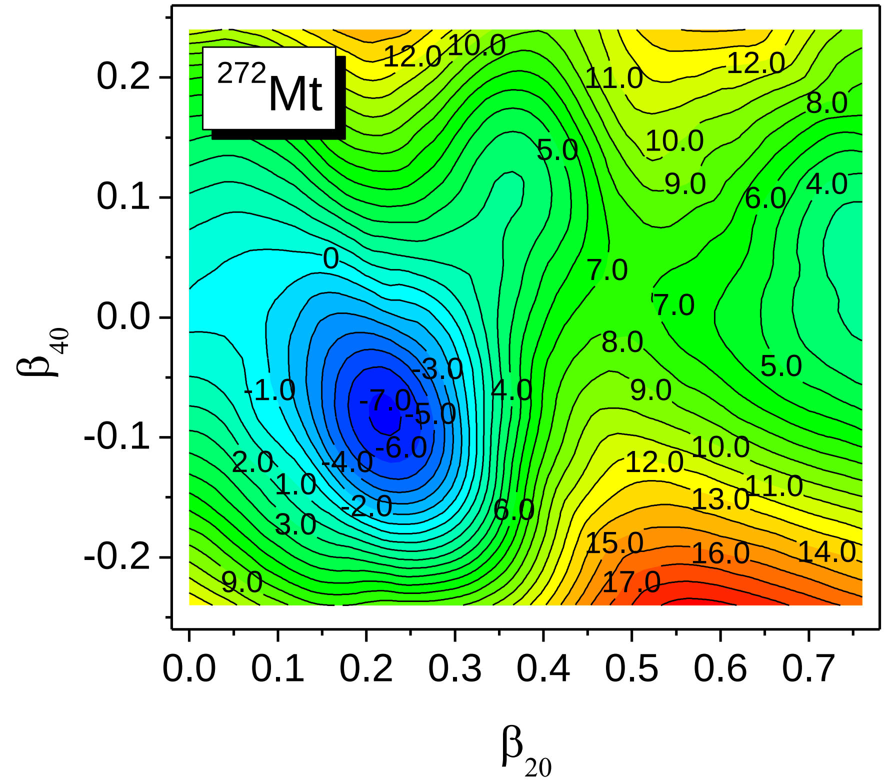


Fig. 17. Fission hindrance factors of odd-mass isotopes with experimentally assigned configuration (spin and parity) of the fissioning states

$$HF(Z,N) = T_{SF,exp}(Z,N)/T_{ee}(Z,N),$$





G.S. configuration:
P:11/2+ [6 1 5]
N:13/2- [7 1 6]

Fixing the g.s. configuration rises the barrier by 6 MeV.
Even if configuration is not completely conserved, a substantial increase in fission half-life is expected.

TABLE I: Fission halflives and hindrance factors for the K-isomers and ground states in the first well.

Nucleus	K^{π}	T_{sf}(g.s.)	T_{sf}(izo)	HF= T_{sf}(izo)/T_{sf}(g.s.)
^{250}No [a]	(6^{+})	3.7 μs	$> 45\mu$s	> 10
^{254}No [b]	8^{-}	3×10^{4} s	1400 s	$\approx \frac{1}{20}$
^{254}Rf [c]	(8^{-})	23 μs	$> 50\mu$s	> 2
	(16^{+})		$> 600\mu$s	> 25

[a] D. Petersen et al., Phys. Rev. C 73, 014316 (2006), F. P. Hessberger, Eur. Phys. J. A 53.
[b] F. P. Hessberger et al., Eur. Phys. J. A 43, 55 (2010).
[c] H. M. David et al., PRL 115, 132502 (2015).

Isomers in the first well

In theoretical models:
odd nucleus – one blocked state
isomer – at least two blocked states

TABLE II: Excitation energies and fission halflives of shape isomers (ground states in the second well), of the excited (probably K-isomeric) states there [a] and the hindrance factors HF= $T_{sf}(\text{izo})/T_{sf}(\text{g.s.})$.

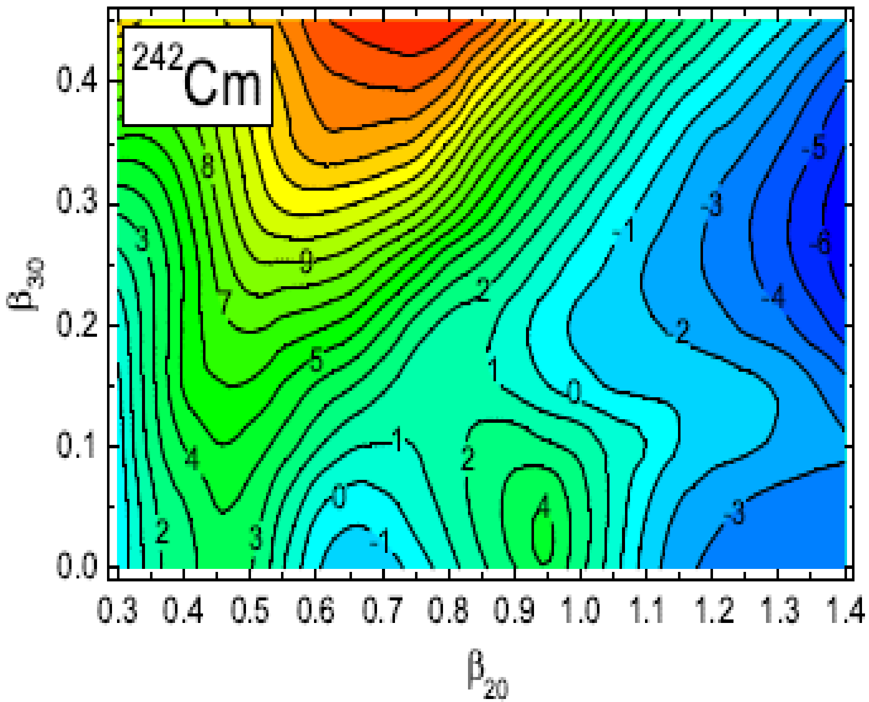
Nucleus	$E(g.s.)$	T_{sf}(g.s.)	E_{izo}	T_{sf}(izo)	HF
^{236}Pu	3.0	37 ns	4.0	34 ns	≈ 1
^{237}Pu	2.6	85 ns	2.9	1.1 μs	
^{238}Pu	2.4	0.6 ns	3.5	6 ns	10
^{239}Pu	3.1	7.5 μs	3.3	2.6 ns	
^{240}Pu	2.2(?)	37 ns			
^{241}Pu	2.2	21 μs	2.3	32 ns	
^{242}Pu	∼2.0	3.5 ns	?	28 ns	8
^{243}Pu	1.7	45 ns			
^{244}Pu	?	0.4 ns			
^{245}Pu	2.0	90 ns			

Isomers in the second well

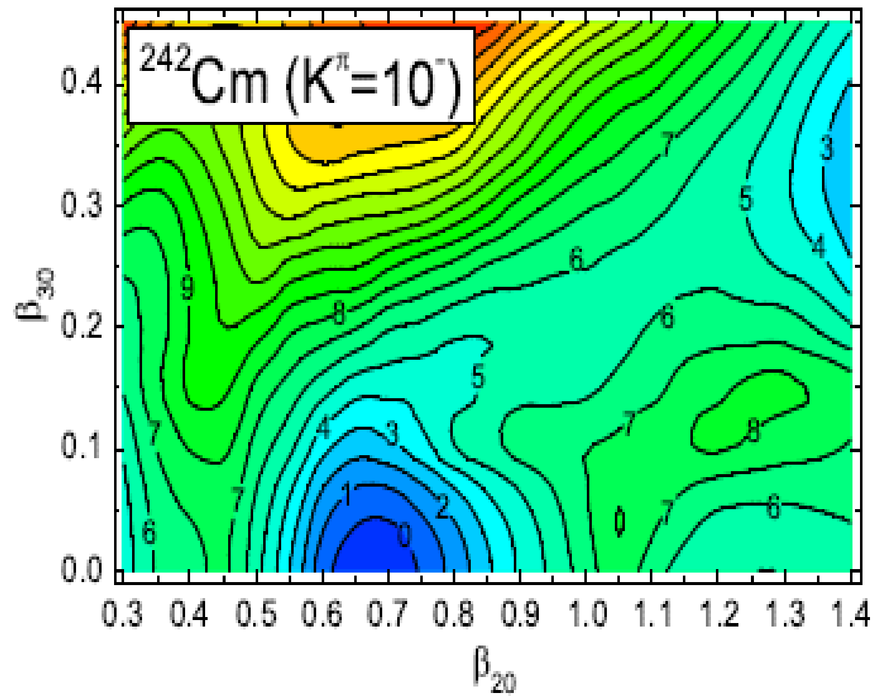
^{237}Am	2.4	5 ns			
^{238}Am	~2.5	35 μs			
^{239}Am	2.5	163 ns			
^{240}Am	3.0	0.94 ms			
^{241}Am	~2.2	1.2 μs			
^{242}Am	2.2	14 ms			
^{243}Am	2.3	5.5 μs			
^{244}Am	2.8	0.9 ms	?	~6.5 μs	
^{245}Am	2.4	0.64 μs			
^{246}Am	~2.0	73 μs			
^{240}Cm	~ 2.0	10 ps	~3.0	55 ns	550
^{241}Cm	~ 2.3	15.3 ns			
^{242}Cm	~ 1.9	40 ps	~2.8	180 ns	4500
^{243}Cm	1.9	42 ns			
^{244}Cm	~ 2.2	< 5 ps	~3.5	> 100 ns	> 20000
^{245}Cm	2.1	13.2 ns			

[a]B. Singh, R. Zywina, and R. Firestone, Nuclear Data Sheets 97 241 (2002).

Barrier 4 MeV higher
and much longer – exp. HF
may come from EM decay



Minimization over possible configurations.

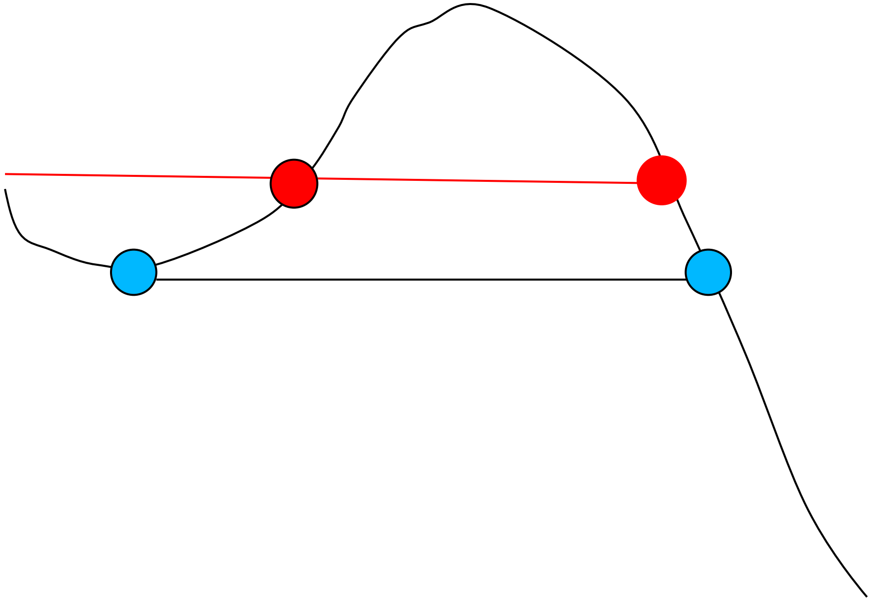


Keeping configuration fixed:

$$\nu 11/2^{+}\ \nu 9/2^{-}$$

(unique candidate)

Remark I



Fission half-lives for isomers do not shorten as suggested by this picture, so the barrier for an isomer must probably rise with respect to that for the g.s.

Remark II

Assuming WKB, for 5 nuclei one can obtain from log(HF) ΔS(odd-even) both at the I-st and II-nd minimum (in units of hbar):

Nucleus	ΔS(I)	ΔS(II)
239Pu	5.80	4.26
241Pu	6.71	4.34
241Am	5.81	3.93
243Am	6.13	5.32
243Cm	5.48	4.0

The barrier for the second minimum is smaller by the excitation of the shape isomer.
Does ΔS(odd-even) come solely from the II-nd barrier?

Ideas:

-Partial release of K quantum number – first barriers are often triaxial;

-Consider the minimization of S allowing the pairing gap to vary freely [L.G. Moretto and R.P. Babinet Phys. Lett. 49B, 147 (1974)]. K. Pomorski & Lublin group found that this decreases the action. Then Yu. A. Lazarev showed in a simple model [Phys. Scripta 35, 255 (1987)] that the action minimization with respect to the gap would reduce (a desired outcome) fission hindrance for odd-A nuclei and isomers.

Caveats:

- The cranking inertia was used in S;
- The gap is related to the Hamiltonian and should be determined by the dynamics before the action is calculated.

Inertia parameter

Question: How to obtain the mass parameter B_{ij}?

Adiabatic approximation leads to the following formula:

$$B_{ij} = 2\hbar^2 \sum_k \frac{\langle k|\partial/\partial q_i|0\rangle \, \langle 0|\partial/\partial q_j|k\rangle}{E_k - E_0}$$

where $|0>$ denotes the ground state. Since we know, that in a nucleus pairing correlations play an important role, we write the ground state of an odd nucleus in a BCS form:

$$|0>= a^+_{\mu_0} \prod_{\mu \neq \mu_0} (u_\mu + v_\mu a^+_\mu a^+_{\bar{\mu}})|vac>$$

The state μ_0 occupied by an odd (unpaired) nucleon is blocked for pairing correlations!

Using BCS wavefunction as a ground state of an odd nucleus we obtain the final formula for the mass parameter:

$$
\begin{aligned}
B_{q_i q_j} = 2\hbar^2 \Bigg[& \sum_{\mu,\nu\neq\nu_0} \frac{\langle\mu|\,\partial_{q_i}\hat{H}\,|\nu\rangle \langle\nu|\,\partial_{q_j}\hat{H}\,|\mu\rangle}{(E_\mu + E_\nu)^3} (u_\mu v_\nu + u_\nu v_\mu)^2 \\
& + \frac{1}{8} \sum_{\nu\neq\nu_0} \frac{(\tilde{\varepsilon}_\nu(\partial_{q_i}\Delta) - \Delta(\partial_{q_i}\tilde{\varepsilon}_\nu))\left(\tilde{\varepsilon}_\nu(\partial_{q_j}\Delta) - \Delta(\partial_{q_j}\tilde{\varepsilon}_\nu)\right)}{E_\nu^5} \Bigg] \\
& + 2\hbar^2 \sum_{\nu\neq\nu_0} \frac{\langle\nu|\,\partial_{q_i}\hat{H}\,|\nu_0\rangle \langle\nu_0|\,\partial_{q_j}\hat{H}\,|\nu\rangle}{(E_\nu - E_{\nu_0})^3} (u_\nu u_{\nu_0} - v_\nu v_{\nu_0})^2
\end{aligned}
$$

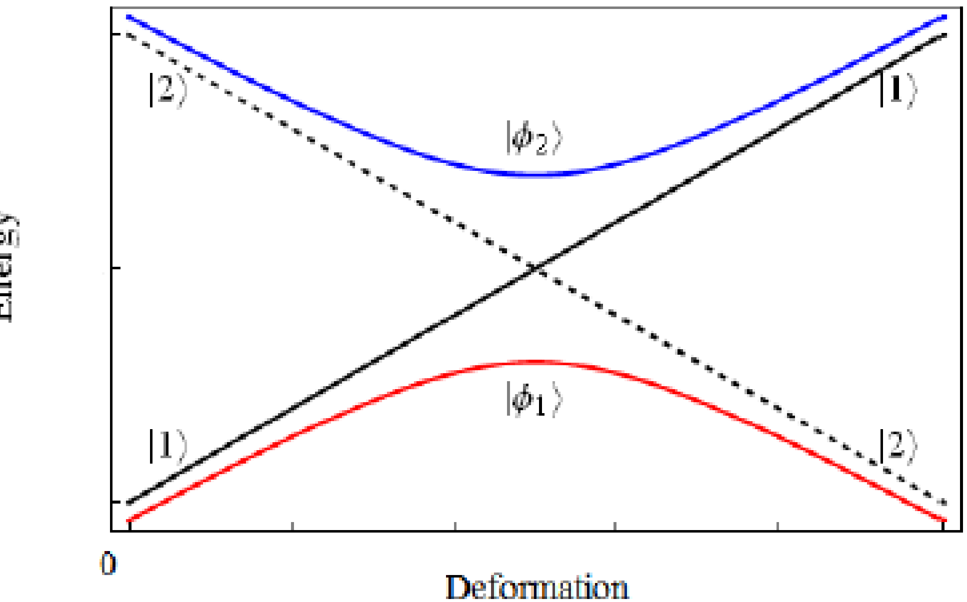
where $E_\mu = \sqrt{\tilde{\varepsilon}^2 + \Delta^2}$, $\tilde{\varepsilon} = \varepsilon - \lambda$ and λ - Fermi energy.

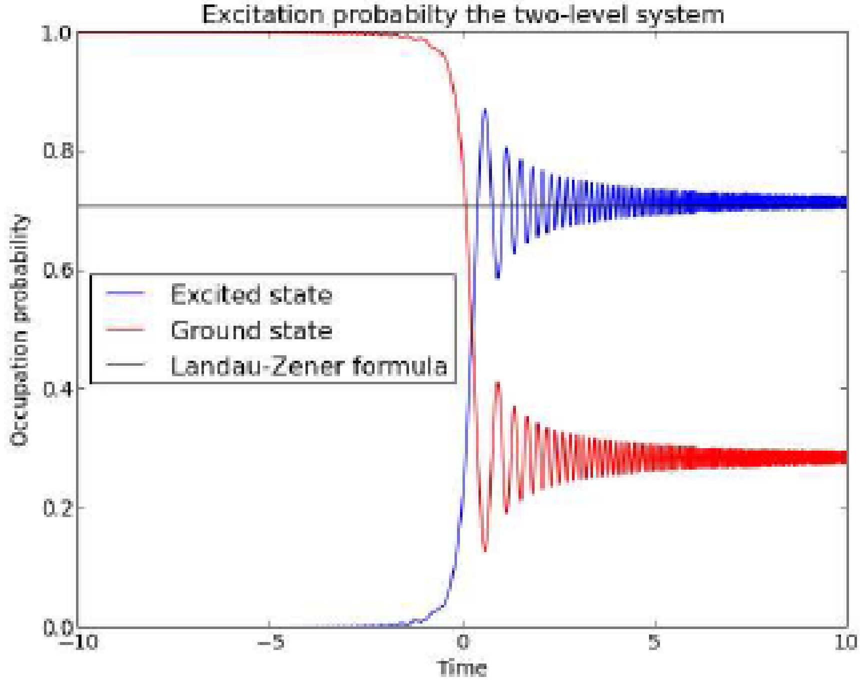
Problems:

- if another state comes close to the blocked state ν_0 then mass parameter explodes!
- if the blocked state ν_0 lies higher in energy than other state ν one gets negative values of mass parameter!

Landau-Zener transition

(No hope for a general velocity-independent mass.)





If the system is initially ($t_i = -\infty$) in the state $|\phi_1>$ the probability, that it finds itself in the state $|\phi_2>$ at $t_f = +\infty$ is given by the Landau-Zener formula:

$$P_{|\phi_2>}(t \to +\infty) = exp\left(\frac{-2\pi}{\hbar}\frac{V^2}{\dot{q}\frac{\partial}{\partial q}(E_2 - E_1)}\right)$$

Instanton method

In field theory: S. Coleman, Phys. Rev. D 15 (1977) 2929
In nuclear mean-field theory:
S. Levit, J.W. Negele and Z. Paltiel, Phys. Rev. C22 (1980) 1979

Reformulation & Connection to other approaches to the Large Amplitude Collective Motion:
J. Skalski, PRC 77, 064610 (2008).

The main idea: even if there is no mass, there is action.

A consequence: the requantization of the collective motion may be sometimes meaningless.

The dynamics of the many-body fermion system can be described, within mean field approximation, by the time-dependent Hartree Fock (TDHF) equations:

$$i\hbar\partial_t\psi_k = \hat{h}(t)\psi_k(t)$$

where $\hat{h}[\psi^*(t), \psi(t)]\psi_k(t) = \delta\mathcal{H}/\delta\psi_k^*(t) \Rightarrow$ nonlinear dependence of $\hat{h}$ on ψ_k.

Properties:

- $< \psi_l|\psi_k >= const,$
- Energy $\mathcal{H} = const.$

Because of the 2nd property TDHF equations cannot be directly used to describe fission process, one has to transform them to imaginary time i.e. $t \rightarrow -i\tau$. Under this transformation $\psi \rightarrow \psi(x, -i\tau) = \phi(x, \tau)$ and $\psi^* \rightarrow \psi^*(x, -i\tau) = \phi^*(x, -\tau)$.

After transformation of the TDHF equations to the imaginary time we obtain:

$$\hbar \frac{\partial \phi_k(\tau)}{\partial \tau} = -\hat{h}(\tau)\phi_k(\tau)$$

where $\hat{h}(\tau) = \hat{h}[\phi^*(-\tau), \phi(\tau)]$.

Since we require our solution to be periodic, i.e. $\phi_k(-T/2) = \phi_k(T/2)$, we add the periodicity fixing term $\epsilon_k \phi_k$ obtaining the instanton equations:

$$\boxed{\hbar \frac{\partial \phi_k(\tau)}{\partial \tau} = -(\hat{h}(\tau) - \epsilon_k)\phi_k(\tau)}$$

The action of an instanton can be calculated in the following way:

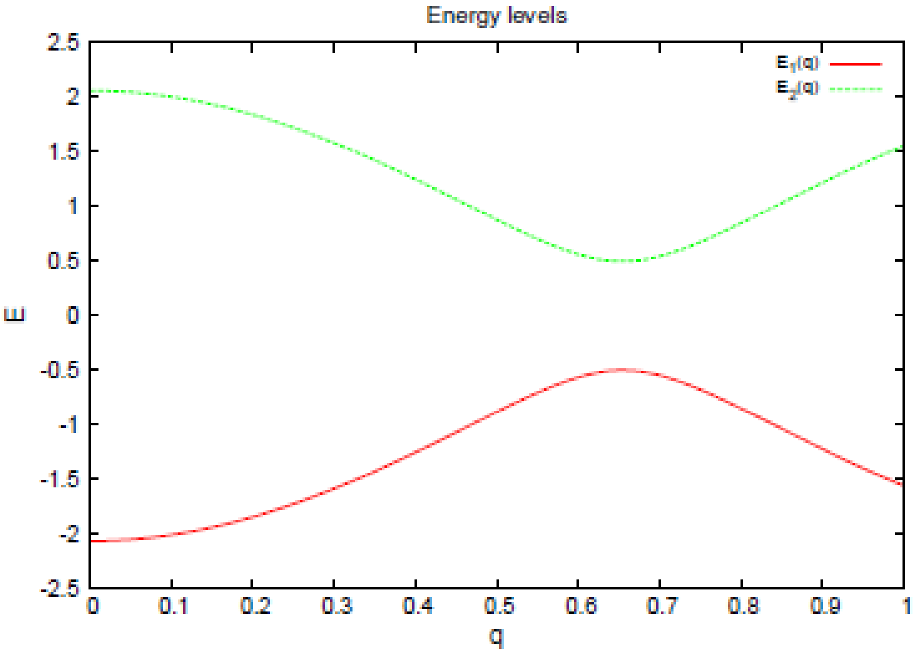
$$\boxed{S = \hbar \int_{-T/2}^{T/2} d\tau \sum_k \left\langle \phi_k(-\tau) \middle| \partial_\tau \phi_k(\tau) \right\rangle}$$

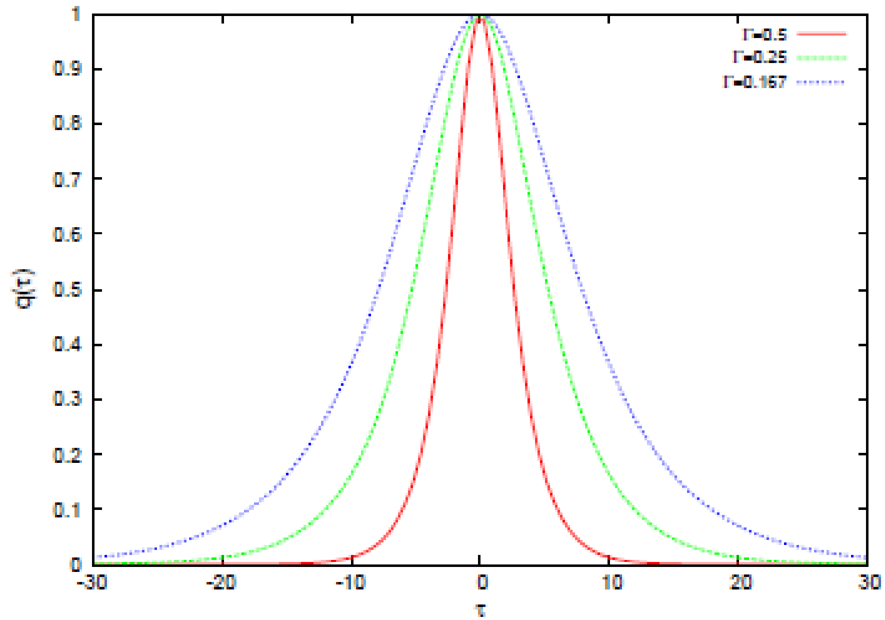
Approximation: We replace the selfconsistent potential in the hamiltonian $\hat{h}(\tau)$ by the phenomenological Woods-Saxon potential.

collective velocity $\dot{q}$ must be provided

$$B_{even}(q)\dot{q}^2 = 2(V(q) - E)$$

Simple 2-level model



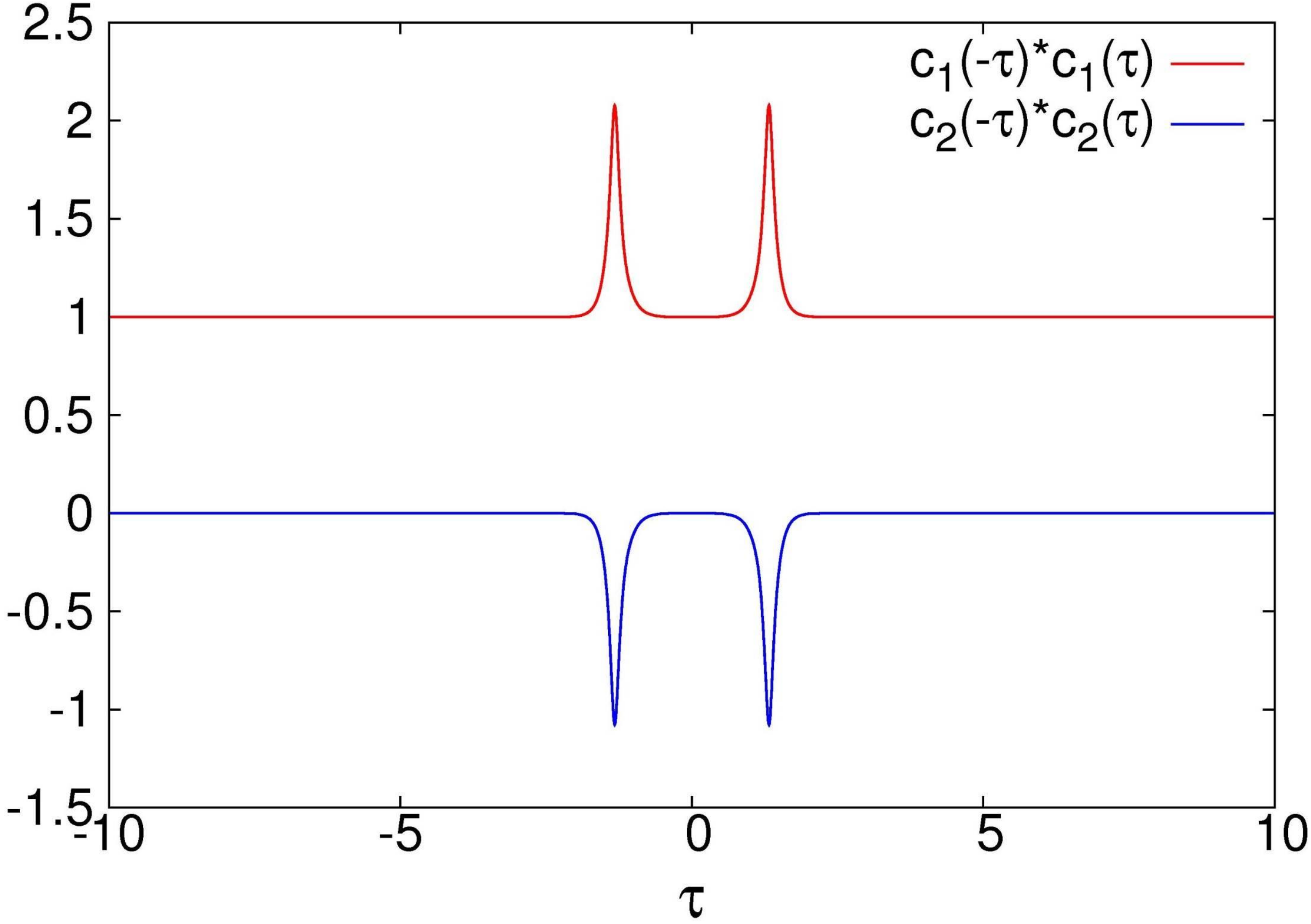


$$\hat{H}(q(\tau))\,\phi_{1,2}(q(\tau)) = E_{1,2}(q(\tau))\,\phi_{1,2}(q(\tau)) \qquad q(\tau) = \frac{q(0) - q(-T/2)}{cosh(\Gamma\,\tau)} + q(-T/2)$$

Quasi-occupations

$$\phi_i(\tau) = \sum_\mu C_{\mu i}(\tau)\psi_\mu(q(\tau)),$$

$$p_{\mu i}(\tau) = C^*_{\mu i}(-\tau)C_{\mu i}(\tau)$$



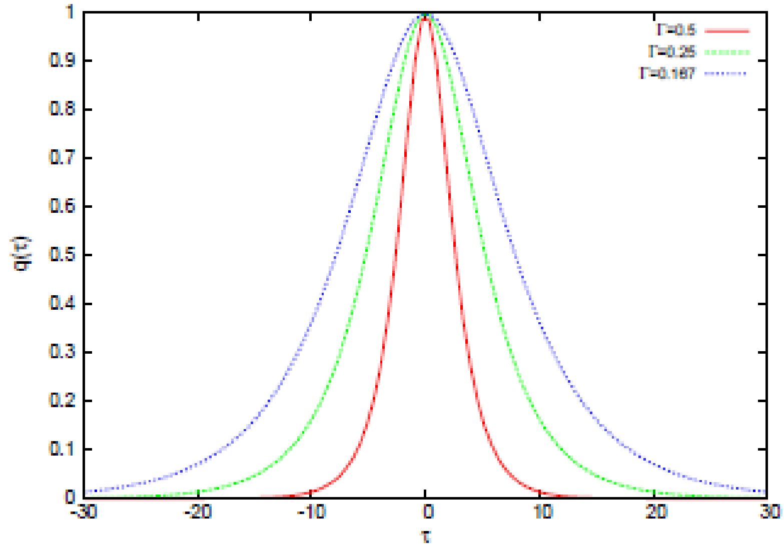
Calculations of the action

Action for the instanton:

$$S_{inst} = \hbar \int_{-T/2}^{T/2} d\tau \left\langle \psi(-\tau) \middle| \partial_\tau \psi(\tau) \right\rangle$$

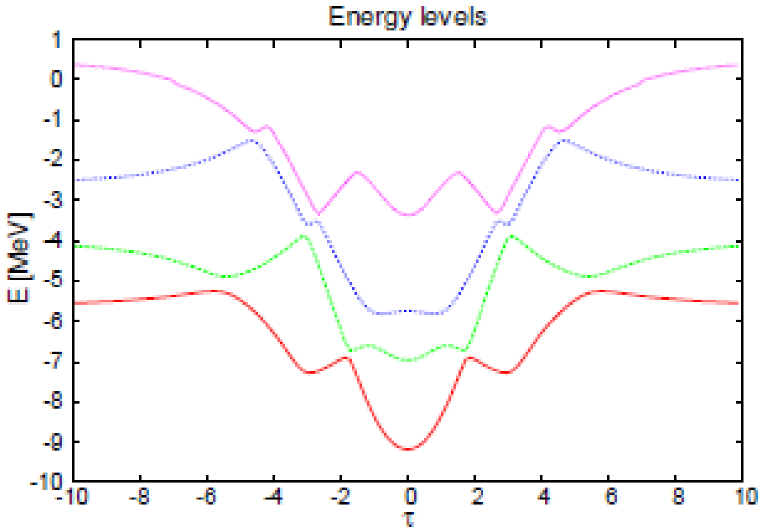
Adiabatic action:

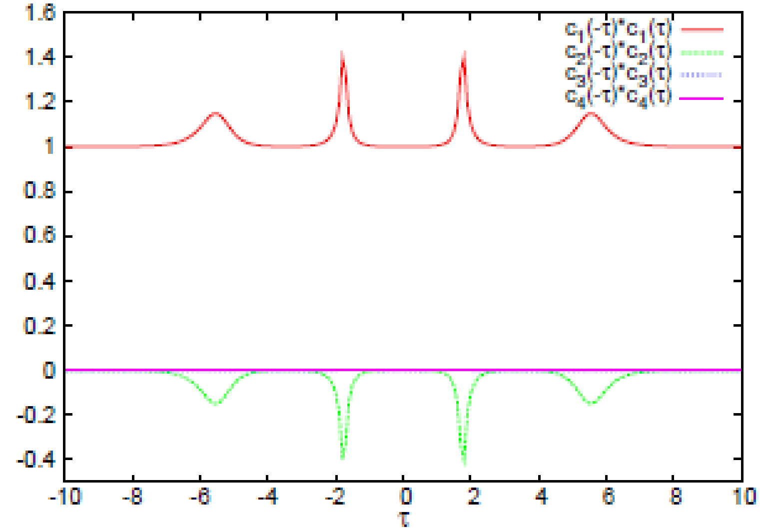
$$S_{adiab} = 2\hbar^2 \int_{-T/2}^{T/2} d\tau \; \dot{q}^2 \frac{|\langle \phi_2 | \partial_q \phi_1 \rangle|^2}{E_2 - E_1}$$



	$\Gamma = 0.5$	$\Gamma = 0.25$	$\Gamma = 0.167$	$\Gamma = 0.5$	$\Gamma = 0.25$	$\Gamma = 0.167$
	$V_{int} = 1$			$2V_{int}$		
$S_{inst}/\hbar$	1.183	0.770	0.569	0.398	0.218	0.149
$S_{adiab}/\hbar$	2.015	1.007	0.672	0.459	0.229	0.152

More realistic case: four $1/2^+$ states taken from the deformed Woods-Saxon potential for Z=109, N=163



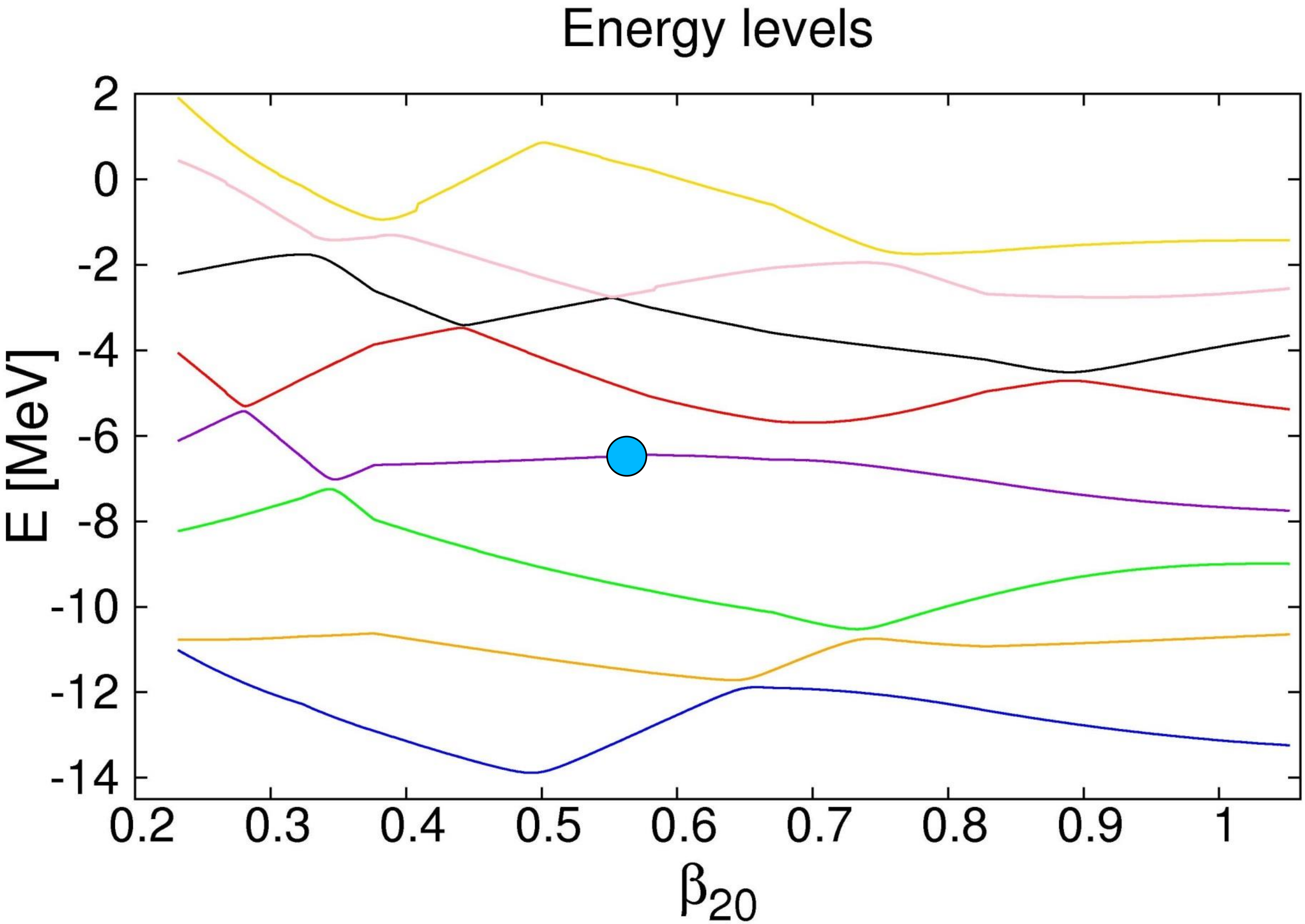


Instanton vs adiabatic action (1st state):

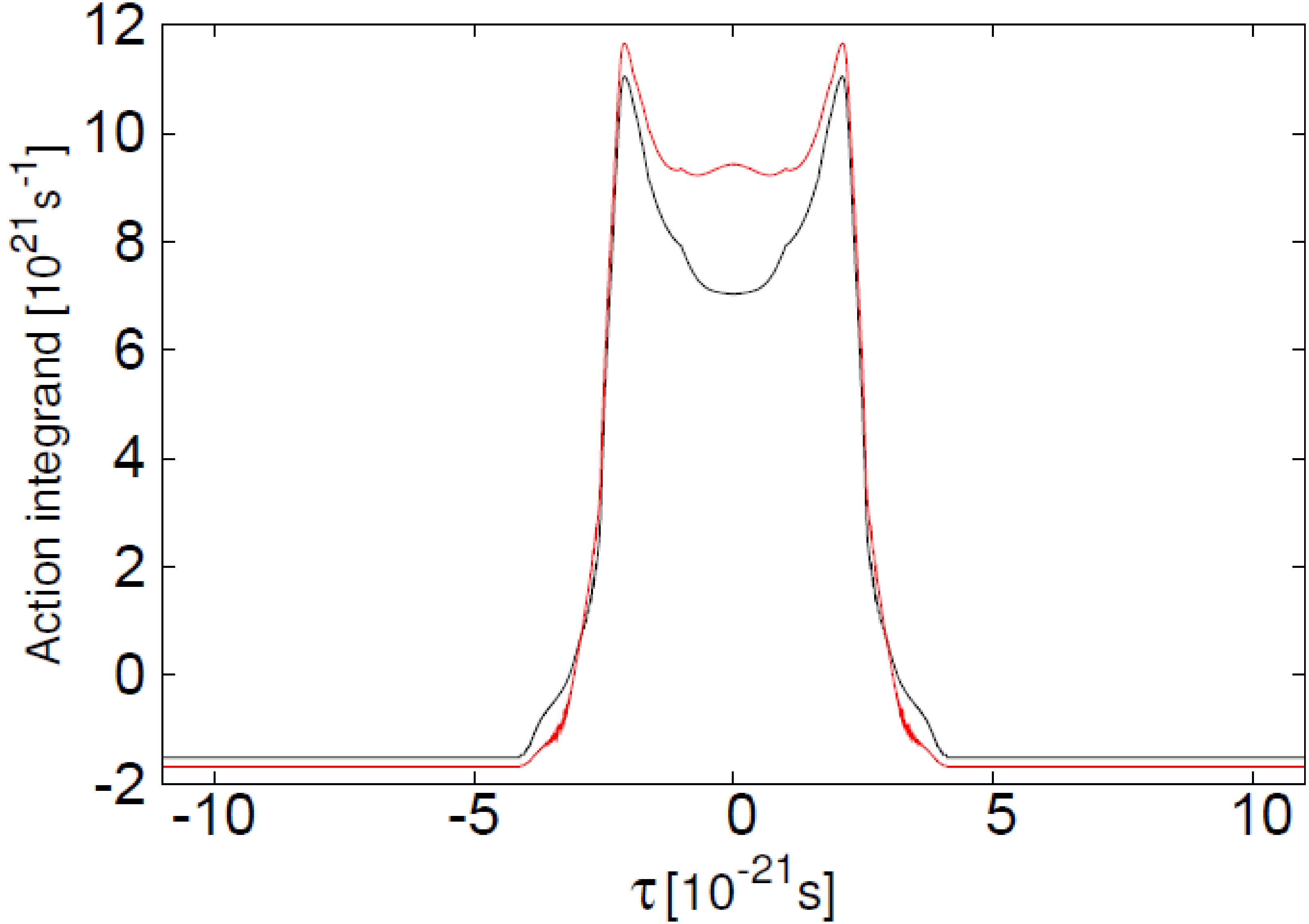
$\hbar\dot{q}_{max}$ [MeV]	$S_{inst}/\hbar$	$S_{adiab}/\hbar$
0.14	2.6818	55.048
0.09	2.4892	36.699
0.06	2.3492	25.689

By the comparison of both action values we can see, how far from the adiabaticity condition we actually are!

Neutron 3/2+ levels along the axially-symmetric fission barrier
Z=109, N=163



action integrands for six (black line) and seven (red line) neutrons



Pairing is important → ImTDHFB; instanton with a smallest action defines the decay rate.

$$\hbar\partial_\tau \begin{pmatrix} A_k(\tau) \\ B_k(\tau) \end{pmatrix} + \begin{pmatrix} \hat{t}+\hat{\Gamma}(\tau), & \hat{\Delta}(\tau) \\ -\hat{\Delta}^*(-\tau), & -(\hat{t}+\hat{\Gamma}(-\tau))^* \end{pmatrix} \begin{pmatrix} A_k(\tau) \\ B_k(\tau) \end{pmatrix} = E_k \begin{pmatrix} A_k(\tau) \\ B_k(\tau) \end{pmatrix}$$

$$S = -\frac{1}{2}\int d\tau Tr[A^+(-\tau)\partial_\tau A(\tau) + B^+(-\tau)\partial_\tau B(\tau)].$$

Non-selfconsistent case: if one takes s.p. energies for t+Γ and diagonal Δ (typical pairing gap), then including time-evolution of the adiabatic basis one has to solve iteratively, self-adjusting Δ as a function on [-T/2,T/2].

Thus, there is a consistent dynamical equation which determines both Δ and action S.

Conclusions

- Experimental data suggest a mechanism for fission hindrance in both odd-A nuclei and isomers.
- Such states can have longer fission half-lives in the SHN.
- The pairing + specialization energy (configuration – preserving) mechanism seems too strong.
- The description of fission for odd-A nuclei and isomers is unsatisfactory – it lacks a sound principle.
-The instanton method adapted to the mean-field formalism may provide a basis for the minimization of action.
- The preliminary, non-selfconsistent studies indicate that
a) the action is well defined for an arbitrary path,
b) a contribution to S from one nucleon is moderate.

The work on paired systems and inclusion of the selfconsistency lies ahead.