# Coulomb excitation as a probe of nuclear structure

**Douglas Cline**

Department of Physics and Astronomy
University of Rochester, Rochester, New York, USA

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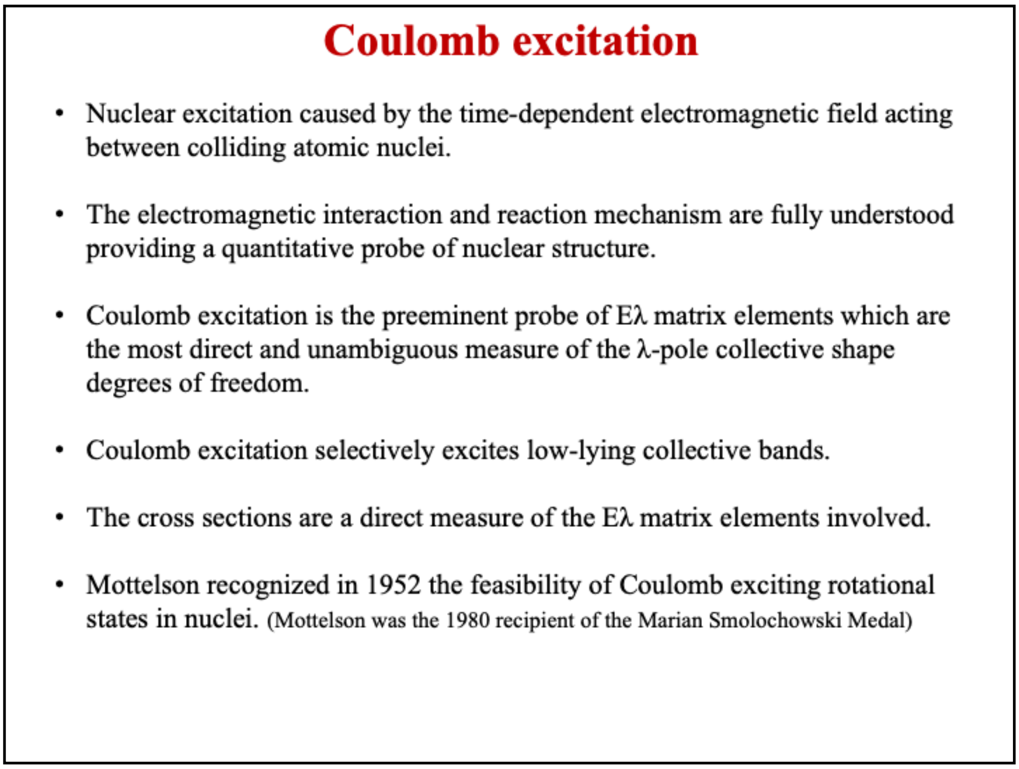
It is tremendous honor to receive the Marian Smoluchowski Medal and I am deeply indebted to everyone involved for including me in the list of distinguished laureates of this award.

This talk entitled “Coulomb excitation as a probe of nuclear structure” will describe collaborative work developed at Rochester plus Warsaw that led to the Warsaw Heavy-Ion Laboratory becoming a world-leading centre in this field of nuclear structure.

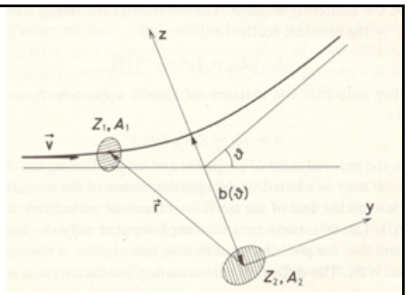
The nuclear many-body quantal system exhibits a complicated and fascinating interplay of single-particle and collective shape degrees of freedom. Collective motion is a dominant and ubiquitous feature of nuclear structure. Nuclear fission is one well-known example where collective motion deformation leads to fragmentation. Nuclear spectroscopy exhibits many other manifestations of the dominant role of collective shape degrees of freedom in nuclei. For example quadrupole-deformed rotational bands structures are a dominant feature of the low-lying spectra in nuclei and the microscopic properties of such nuclei can be described in terms of nucleons bound in a rotating deformed potential well. Superdeformed bands, with about twice the quadrupole deformation of the normal rotational bands are observed in many nuclei away from shell closures while collective quadrupole and octupole vibrational modes also are manifest in many nuclei. Rotational bands coexist with shell structure even in closed shell nuclei.

As a consequence, a knowledge of collective shape degrees of freedom in nuclei is pivotal to understanding nuclear structure.




- Coulomb excitation is nuclear excitation caused by the time-dependent electromagnetic field acting between colliding atomic nuclei.
- The electromagnetic interaction and reaction mechanism both are fully understood providing a quantitative probe of nuclear structure.
- Coulomb excitation is the preeminent probe of Eλ matrix elements providing the most direct and unambiguous measure of the λ-pole collective shape degrees of freedom because:
- Coulomb excitation selectively excites low-lying collective bands.
- The cross sections are a direct measure of the Eλ matrix elements involved.
- Mottelson recognized the feasibility of exciting rotational states in deformed nuclei in 1952.
- Ben Mottelson was the 1980 recipient of the Marian Smolochowski Medal

# Coulomb excitation assumptions




## 1) Interaction is purely electromagnetic

The Coulomb-nuclear interaction is $\leq 10^{-3}$ effect for heavy ion collisions if the semiclassical distance of closest approach ,

$$d \geq [1.25(A_T^{1/3} + A_P^{1/3}) + 5] \text{ fm}$$

(D. Cline, Nuclear Physics A133 (1969) 445)

## 2) Semi-classical approximation for orbit

a) Require size of wave packet to be much smaller than the dimensions of the classical hyperbolic trajectory. That is, the Sommerfeld parameter $\eta >> 1$

$$\eta = \frac{b}{2\lambda\!\!\!^{-}} = Z_p Z_T \frac{e^2}{\hbar v} >> 1$$

For typical heavy-ion Coulomb excitation $50 \leq \eta \leq 500$

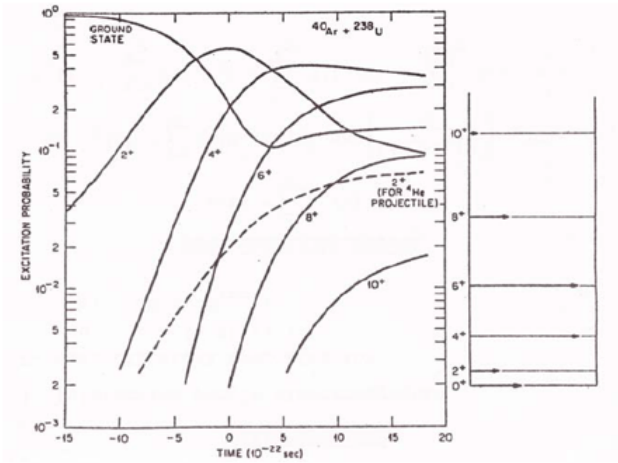
b) Classical hyperbolic Rutherford orbit assumes the energy loss ΔE is small compared with incident energy E.

$$\frac{\Delta E}{E} << 1$$

For typical heavy-ion Coulomb excitation $\Delta E/E \sim 10^{-3}$

- The primary assumption underlying Coulomb excitation is that only the well-known electromagnetic interaction is involved in the two-body reaction. For heavy-ion induced Coulomb excitation it was shown that Coulomb-nuclear interaction effects are less than $10^{-3}$ when the classical distance of closest approach exceeds the sum of the nuclear radii plus 5fm.
- For heavy-ions this simple-minded criterion can be slightly relaxed by staying forward of the grazing angle corresponding to this separation distance.
- For heavy-ion induced Coulomb excitation exact quantal coupled-channel calculations are impractical because: (a) the long-range of the Coulomb force requires including thousands of partial waves due to the small wavelength relative to the separation between the colliding nuclei, and (b) Almost one hundred strongly-coupled states excited can be involved.
- Fortunately the semiclassical approximation is applicable and greatly simplifies the calculation. This approximation assumes that the nuclear excitation is caused by the time-dependent electromagnetic field acting on the colliding nuclei as the projectile moves along a classical hyperbolic trajectory in the Coulomb field of the target nucleus. The semiclassical approximation is satisfied if (a) the wavelength of the projectile ƛ is much smaller than the distance of closest approach, that is, the Sommerfeld parameter. and (b) the ratio of energy transfer to total kinetic energy is small. Thus the use of a classical trajectory is well obeyed.
- For heavy-ion induced Coulomb excitation η typically ranges from 30 to 500 while $\Delta E/E \sim 10^{-3}$ .

**Semiclassical Coulomb excitation prediction of the time evolution for excitation of the ground band of $^{238}U$ by backscattered 140 MeV $^{40}Ar$**




- Multistep Coulomb excitation of collective bands dominates using high-Z ions.
- Coulomb excitation ~ 3 x $10^{-21}$ sec
- γ-ray deexcitation ~ $10^{-13} \rightarrow 10^{-12}$ sec
- Can treat the excitation and γ-ray deexcitation as decoupled sequential processes.

- The semiclassical approximation makes it possible to elucidate the time evolution of the population of excited states as the projectile moves along the classical hyperbolic trajectory.
- The dashed line shows that excitation using backscattered alpha particles exhibits a gradual increase in excitation probability of the 2+ state.
- The figure also shows the population probabilities for backscattering of 140MeV $^{40}Ar$ ions by a $^{238}U$ target result in multistep excitation. That is, multistep excitation is dominant for heavy ions but not for alpha particles.
- The zero of time corresponds to the classical turning point and most excitation occurs within ±15 fm of the zero. The final population distribution is shown on the right. Note that the time scale for excitation is ~$3x10^{-21}$s which is about $10^8$ times shorter than typical lifetimes for γ-ray decay which are $10^{-13}$ -$10^{-12}$s. Thus it is valid to treat the excitation and de-excitation as separable and sequential processes.

## Genesis of Rochester-Uppsala-Warsaw collaboration

1965: Winther and de Boer wrote the semiclassical Coulomb excitation computer code **COULEX** which enabled quantitative analysis of multiple Coulomb excitation data.

1966-78: Coulomb excitation measurements of excited state static quadrupole moments were performed at Rochester. Developed the heavy-ion γ-ray coincident detector technique.

1966-69: Wrote the Coulomb excitation γ-ray yield code **CEGRY** for use in combination with **COULEX** to calculate the recoil-ion γ-ray coincidence rates. (Cline, Lesser)

1975-76: Sabbatical leave at LBNL. Performed multiple Coulomb excitation studies using high-Z beams that became available from the SuperHILAC (LBNL) and UNILAC (GSI).

1976-82: Used the Rochester tandem plus the SuperHILAC for Coulomb excitation studies of rotational and shape-transitional nuclei, $^{72}$Ge, $^{104}$Ru, $^{110}$Pd, $^{168}$Er, $^{182,184}$W, $^{186,188,190,192}$Os, $^{194}$Pt, $^{248}$Cm.

Faculty visitors; Julian Srebrny (Warsaw)
Lennart Hasselgren (Uppsala)

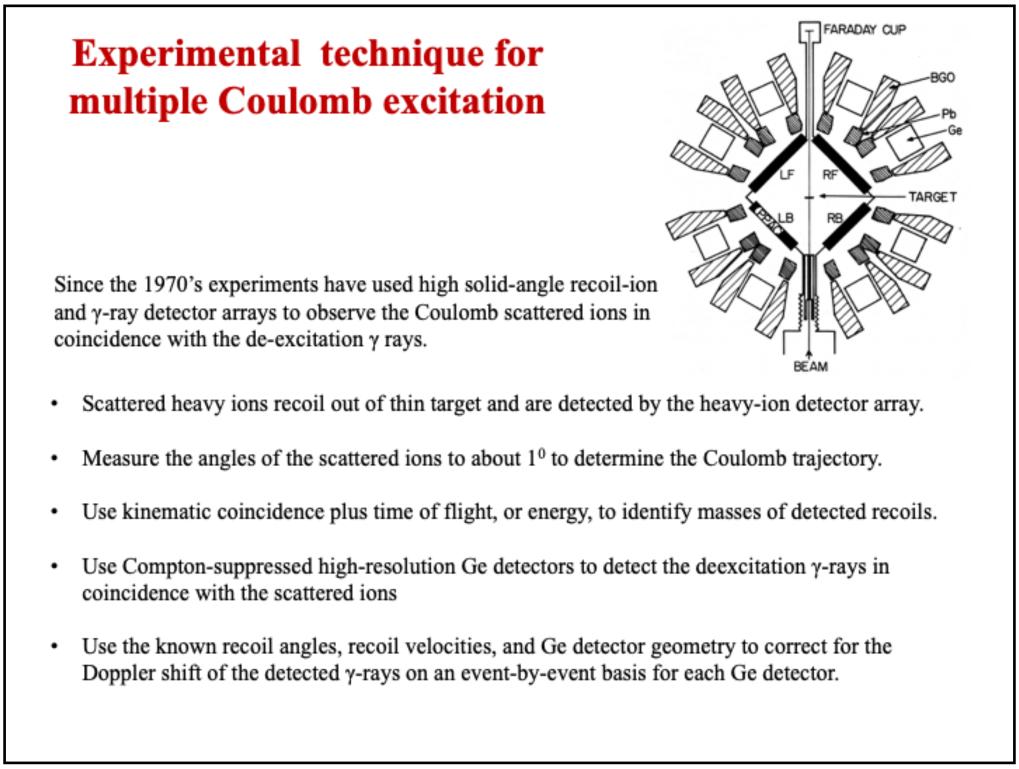
Ph.D. students: Tomasz Czosnyka (Warsaw)
Bohdan Kotlinski (Warsaw)
Ching-Yen Wu (Rochester)

In 1965 Winther and de Boer developed the semiclassical Coulomb excitation computer code COULEX which enabled quantitative analysis of multiple Coulomb excitation data. This development provided the first viable way to fully exploit multiple Coulomb excitation in nuclear physics.

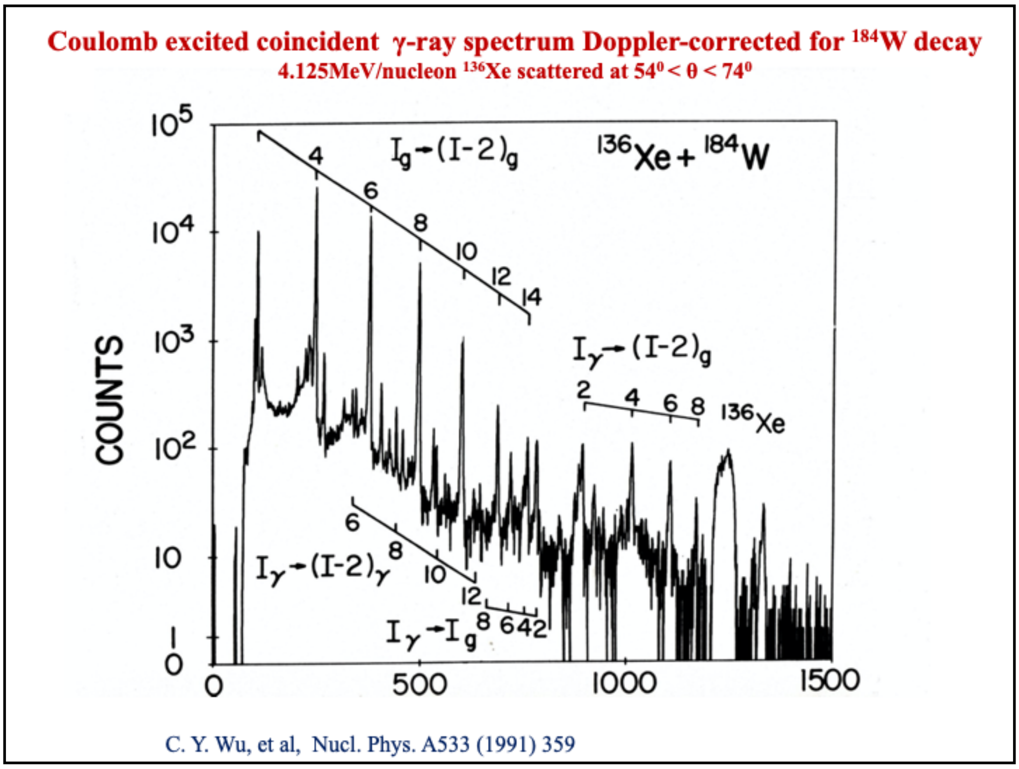
In 1966 we initiated a program to use heavy-ion beams from the new Rochester MP tandem accelerator to exploit the reorientation effect of multiple Coulomb excitation to measure static electric quadrupole moments of excited states. In 1969 we developed a heavy-ion γ-ray coincidence technique to achieve greater sensitivity than achieved using a magnetic spectrometer which we used initially.

Cline and Lesser wrote the γ-ray yield code CEGRY for use in combination with the COULEX code to calculate the recoil-ion γ-ray angle-dependent coincidence rates following Coulomb excitation.

In 1975-76 the LBNL SuperHILAC and GSI UNILAC both achieved the capability to accelerate the heaviest ion beams such as 208Pb. That year I spent a sabbatical year at Berkeley where we established a multiple Coulomb excitation program using high-Z heavy-ion beams. From 1976 – 82 Julian Srebrny, Lennart Hasselgren plus students, used the heaviest-ion beams from the SuperHILAC plus lighter-mass beams from the Rochester tandem accelerator to Coulomb excite a dozen strongly-deformed and shape-transitional nuclei. Julian persuaded two brilliant students from Warsaw, Tomasz Czosnyka and Bohdan Kotlinski, to join the Rochester group. This group pioneered the research program that led to the results presented today.



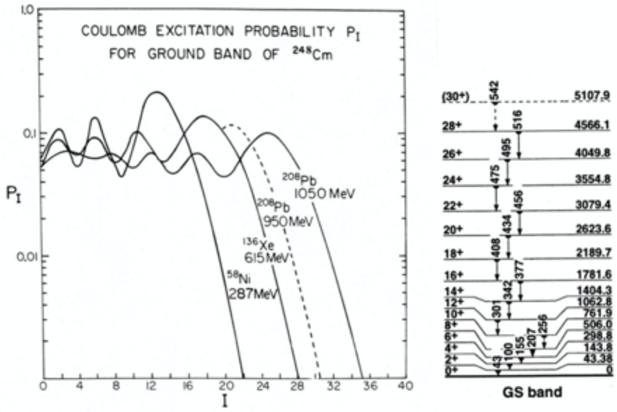

- How are multiple Coulomb excitation experiments performed?
- Since the 1970's we have used large solid angle recoil-ion and γ-ray detector arrays to observe the Coulomb scattered ions in coincidence with the deexcitation γ rays.
- Scattered heavy ions recoil out of thin target and are detected by heavy-ion detectors.
- The angles of the scattered ions are measured to about $1^0$ to determine the Coulomb trajectory.
- Use kinematic coincidence, time of flight, or energy to identify the masses of detected recoils.
- Use Compton-suppressed high-resolution Ge detectors to detect the deexcitation γ-rays in coincidence with the scattered ions
- Using the known recoil angles, recoil velocities, and Ge detector geometry allows correction for the Doppler shift of the detected γ-rays on an event-by-event basis for each Ge detector.




This slide shows the coincident γ-ray spectrum for Coulomb excitation of $^{184}$W by 4.125MeV per nucleon $^{136}$Xe detected at scattering angles between $54^0$ and $74^0$ using the experimental geometry shown previously. The spectrum has been corrected for the Doppler effect on an event-by-event basis assuming the γ-rays are emitted by the $^{184}$W recoil. The resulting energy resolution is 0.5% FWHM.

Note that the deexcitation γ-ray from $^{136}$Xe is Doppler broadened. Making the Doppler correction for the $^{136}$Xe recoil results in a 0.5% resolution for the $^{136}$Xe γ-ray and Doppler broadened peaks for the $^{184}$W. Thus the γ-ray transitions from the two recoiling ions can be unambiguously assigned to the correct recoil product.

**Extraction of Eλ matrix elements from multiple Coulomb excitation data**




- The Coulomb excitation deexcitation γ-ray yields can be calculated using the COULEX plus CEGRY codes if the E2 matrix elements are known.
- The inverse procedure of extracting the E2 matrix elements from Coulomb excitation data is much more challenging because the γ-ray yields for each level depend in a complicated non-linear way on the magnitudes and relative signs of many E2 matrix elements.
- Is it viable to make a least-squares fit of the many Eλ matrix elements for a nucleus assuming the multiple Coulomb excitation γ-ray yield data over-determine the system?

- How do we extract the Eλ matrix elements from the Coulex data?
- For the ground band of $^{248}Cm$ the 19 levels with I ≤ $36^+$ are coupled by 36 E2 matrix elements.
- The calculated probability for Coulomb excitation of the ground band of $^{248}Cm$ for back-scattered heavy ion beams is illustrated.
- Note that the Coulomb coupled equation system is so strong that the excitation probabilities of the ground state up to a classical spin value are roughly constant.
- The Coulomb excitation deexcitation γ-ray yields can be calculated using the COULEX plus CEGRY codes if the E2 matrix elements are known.
- However, the inverse procedure of extracting the E2 matrix elements from Coulomb excitation data is much more challenging because the γ-ray yields for each level depend in a complicated non-linear way on the magnitudes and relative signs of many E2 matrix elements.
- A relevant question; "Is it viable to make a least-squares fit of the 36 Eλ matrix elements to >100 multiple Coulomb excitation γ-ray yield data?"
- In 1979 many experts in the field told me that this was not possible. I disagreed with them because I believed that it is possible to measure sufficient data to over-determine the system and then extract the individual matrix elements.

## Model-independent measurement of Eλ matrix elements from multiple Coulomb excitation data

1) Model-independent manual analysis of $^{110}$Pd using COULEX/CEGRY determined a set of E2 matrix elements that reproduced the data. Could not test the uniqueness of the solution.
(L. Hasselgren and D. Cline, Ettore Majorana Int. Sci. Series, Vol 10 (Ed, F. Iachello,Plenum Press) 1980 p59)

2) Least-square search code GOSIA developed by Czosnyka, Cline, and Wu, 1979-80.

3) GOSIA analysis of $^{110}$Pd data confirmed the results of the manual analysis; 1980.

4) Recoil-distance lifetime measurements for 8 states confirmed the correctness and accuracy of the GOSIA analysis for $^{110}$Pd.
(B. Kotlinski, Ph.D. thesis, Univ. Rochester 1984, B. Kotlinski et al, Nucl. Phys. A503 (1989) 575)

5) Confirmed that many Eλ matrix elements can be measured via multiple Coulomb excitation.

6) GOSIA model-independent analysis of the $^{72}$Ge, $^{104}$Ru, $^{110}$Pd, $^{168}$Er, $^{186-192}$Os, $^{194}$Pt, $^{248}$Cm.
(C.Y. Wu, Ph.D. thesis, Univ. Rochester 1983)
(T. Czosnyka, PhD. Thesis, Warsaw University, 1984)
(B. Kotlinski, Ph.D. thesis, Univ. Rochester 1984)

A real breakthrough was made by Lennart Hasselgren. He is a remarkably well organized person who succeeded in making a model-independent analysis of the Coulomb excitation data for $^{110}$Pd using the codes COULEX plus CEGRY. A careful and detailed analysis, involving use of large complicated tables and requiring one year of effort, located an apparent best solution. The results had scientifically interesting implications. Unfortunately there was no practical way to prove the correctness nor the uniqueness of his solution. Very few people have the skill and patience needed to reproduce such a time-consuming manual analysis. Thus in 1979 Tomasz Czosnyka developed a Coulomb excitation least-squares search code based on COULEX plus CEGRY to provide a practical way for analyzing multiple Coulomb excitation data. This was an extremely challenging task. It required developing sophisticated computational techniques plus use of supercomputers to succeed. The code was named after Tomasz's wife who was not allowed to leave Poland at that time.

The first success of GOSIA was when it was used to analyze the $^{110}$Pd data. This model independent analysis located a unique solution which agreed with the result of the manual analysis mase by Lennart Hasselgren.

Danek Kotlinski used the recoil distance technique following Coulex to directly measure the lifetimes of some of the excited states in $^{110}$Pd and these agreed with the values derived from the GOSIA analysis of the Coulex cross sections.

The thesis work of Kotlinski and Wu involved extensive model-independent and exhaustive analyses for the rotational nucleus $^{168}$Er and the shape-transitional Os-Pt nuclei. All of the above studies showed that a model-independent analysis of Coulomb excitation data can model-independently determine Eλ Matrix elements. The success of this program showed that the naysayers are wrong.

The success of this work stimulated a rapid growth in multiple Coulomb excitation work led by the Rochester, Uppsala, Warsaw groups and later the Liverpool group joined with us. The GSI based groups also were doing similar Coulomb excitation work at the UNILAC in parallel with our collaboration. Initially they developed a Coulomb excitation analysis code but they eventually adopted the more powerful Rochester/Warsaw code GOSIA.

## Coulomb excitation least-squares search code GOSIA

T. Czosnyka, D. Cline, C-Y, Wu

- Semiclassical, experiment-oriented, modular program to support all stages of Coulomb excitation including simulation, and data analysis.
- Perform least-squares fit of 999 matrix elements (E1, E2, E3, E4, M1, M2) coupling 100 states to Coulomb excitation data. E5, and E6 couplings included but not fit.
- Fit to 48,000 γ-ray yields from ≤ 50 independent experiments, plus branching ratios, E2/M1 mixing ratios, lifetimes, and known matrix elements.
- Fast semi-analytic approximations to the coupled-channel Coulomb excitation are used to achieve the speed necessary for least-squares fitting and error calculations.

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|---|---|
| "*Gosia User Manual*" | D. Cline, T. Czosnyka, A.B. Hayes, P. Napiorkowski, N. Warr, C-Y. Wu. |
| GOSIA Wiki: | www-user.pas.rochester.edu/~gosia/mediawiki/ |
| Rochester GOSIA website: | www.pas.rochester.edu/~cline/Gosia/index.html |

Development of the code GOSIA was a crucial development . This experiment-oriented modular program designed to support all stages of Coulomb excitation data analysis. To minimize the computer memory usage the code is internally overlaid, i.e. independent modules share, whenever possible, the same memory locations. The present version of GOSIA, which requires about 1.5MB of memory on a 64-bit machine, can fit up 999 matrix elements (E1,E2,E3,E4,M1,M2) coupling up to 75 nuclear levels to a data set comprising a maximum of 48000 γ-ray yields observed in up to 50 independent experiments as well as other available spectroscopic information such as branching ratios, E2/M1 mixing ratios, mean lifetimes and known matrix elements. The E5 and E6 matrix elements can be included, but not varied. The speed necessary to perform the multidimensional fitting of the matrix elements has been achieved by the use of a semianalytic approximation to the coupled-channel Coulomb excitation formalism, The same approximation is employed to estimate the errors of fitted matrix elements and to provide information concerning the influence of individual matrix elements on the observed data. A further acceleration of both excitation and γ-ray deexcitation calculations is achieved by storing frequently used values, which are independent of the matrix elements, to minimize the computational load when matrix elements are varied. GOSIA can perform various tasks as specified by the user.

## Deriving the physics implications from many measured electromagnetic matrix elements

- Two techniques available for deriving the physics implications from a large set of matrix elements.

1. ***Model-dependent*** **comparison of each individual measured matrix element with macroscopic or microscopic model predictions.**
   Model dependent and the sensitivity to the collective parameters can be ambiguous. Precise.

- Collectivity implies observables are correlated within a collective band.

2) **Use** ***model-independent*** **rotational invariants to project the body-fixed collective properties directly from the observables measured in the laboratory frame.**
   Model-independent method directly projects the collective degrees of freedom from the data. Low-precision.

   (Cline, Proc. Orsay Coll. Inst. Phys. Nucl. 1971)
   (Cline, Flaum, Proc. Int. Conf. Sendai 1972)
   (Kumar, Phys. Rev. Lett. 1972 )

- The development of the experimental techniques to make multiple Coulomb excitation experiments, plus use of the code GOSIA to extract the electromagnetic matrix elements from these data, was a tremendous success that posed a new problem. What is the optimum approach to extract the underlying physics from a large set of electromagnetic data?
- One approach is to make a one-on-one comparison with corresponding values predicted by nuclear structure models for each of the hundreds of individual measured matrix elements. Typically the sensitivity of such model-dependent comparisons to the collective and single-particle structure is not readily apparent. The physics implications are more transparent if the data can be directly related to the parameters of the model.
- Collectivity implies correlated observables for any band of states. For example a rotational band implies that in the body-fixed frame both the inertia tensor, and the $E\lambda$ shape tensor of the rotational band, should be correlated from state to state. Thus a more intuitive approach is to express the data in terms of the collective shape degrees of freedom. This can be performed using the rotational invariant technique we had developed in 1972.

## Quadrupole collective model parameters

**Density shape contours**

$$R(\theta,\phi,\rho,t) = R(\rho)\left[1 + \sum_{\mu=-2}^{+2}(-)^{\mu}\alpha^2_{-\mu}(\rho,t)Y^2_{\mu}(\theta,\phi)\right]$$

Quadrupole deformation spherical tensor $\alpha^2_\mu$

Intrinsic frame deformation tensor

$$\alpha^2_0 \equiv \beta\cos\gamma$$
$$\alpha^2_2 = \alpha^2_{-2} \equiv \frac{1}{\sqrt{2}}\beta\sin\gamma$$
$$\alpha^2_1 = \alpha^2_{-1} = 0$$

**Rotational invariants**

Scalar products $(\alpha^2 \times \alpha^2)^0$ and $(\alpha^2 \times \alpha^2 \times \alpha^2)^0$ are rotational invariants.

(K. Kumar, 1971 private communication)

**Electric quadrupole tensor** E(2,μ)

Intrinsic frame E2 spherical tensor

$$E(2,0) = Qcos\delta$$
$$E(2,\pm2) = \frac{1}{\sqrt{2}}Qsin\delta$$
$$E(2,1) = E(2,1) = 0$$

- Collectivity implies correlated observable for states in a collective band. For example a rotational band implies that in the body-fixed frame the inertia tensor and shape are similar for different states of the rotational band.
- For quadrupole deformation the shape of the density contour ρ can be expressed in terms of a quadrupole deformation tensor.
- The intrinsic frame is that for which the 5-component deformation tensor reduces to two parameters plus three Euler angles specifying the orientation of the intrinsic frame. It is usual to express these two parameters in terms of magnitude β and triaxiality parameter γ. A prolate shape has $\gamma=0^0$ while an oblate shape has $\gamma=60^0$ .
- By analogy one can define an instantaneous intrinsic frame for the electric quadrupole tensor which I write in terms of two parameters Q and δ plus the three Euler angles specifying the orientation of the intrinsic frame.
- Within a collective model of the charge density distribution Q is related to β and δ to γ.
- In 1971 Kumar pointed out that scalar products of the deformation tensor are scalars under rotation. I immediately realized that this has broader implications in that scalar products of the E2 spherical tensors are rotational invariants.

## E2 Rotational Invariants

$$\begin{aligned} E(2,0) &\equiv Q\cos\delta \\ E(2,\pm 2) &\equiv \frac{Q}{\sqrt{2}}\sin\delta \\ E(2,\pm 1) &\equiv 0 \end{aligned}$$

$$< S\left|\{E2\times E2\}^0\right| S >= \frac{1}{\sqrt{5}} < S\left|Q^2\right| S >$$

$$< S\left|\{[E2\times E2]^2\times E2\}^0\right| S >= -\sqrt{\frac{2}{35}} < S\left|Q^3\cos 3\delta\right| S >$$

$$< S\left|\{[E2\times E2]^J\times [E2\times E2]^J\}^0\right| S >= \frac{1}{5} < S\left|Q^4\right| S >$$

$$< S\left|\{([E2\times E2]^2\times E2)^J\times [E2\times E2]^J\}^0\right| S >= -\sqrt{\frac{2}{175}} < S\left|Q^5\cos 3\delta\right| S >$$

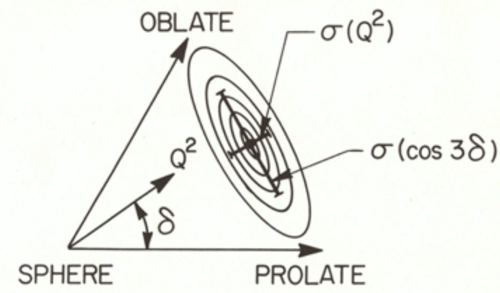
$$< S\left|\{[E2\times E2]^J\times [E2\times E2]^J\times [E2\times E2]^0\}^0\right| S >= \frac{1}{5\sqrt{5}} < S\left|Q^6\right| S >$$

$$< S\left|\{([E2\times E2]^2\times E2)^J\times ([E2\times E2]^2\times E2)^J\}^0\right| S >= \frac{2}{35} < S\left|Q^6\cos^2 3\delta\right| S >$$

The expectation values for a state S of zero-coupled products of E2 operators are scalars that are rotationally invariant, i.e. they are identical in the instantaneous frame defined by the orientation of the symmetry axes of a nucleus, as well as in the fixed laboratory frame. Thus if we can evaluate these rotational scalar invariants in the laboratory frame then this directly gives the expectation values of the quadrupole tensor in the intrinsic frame. Intermediate-state expansions of the rotational invariants can be evaluated as the sums of the products of the reduced E2 matrix elements using the experimental values of these matrix elements. Higher order products of E2 operators can be formed using couplings having various intermediate-spins, thus allowing use of different subsets of E2 matrix elements to evaluate the same rotational invariants. This provides a check of both the validity of a collective picture and self-consistency of the fitted set of the matrix elements.

# E2 Rotational Invariants

D. Cline, Proc. Orsay Coll. Inst. Phys. Nucl. 1971
D. Cline, C. Flaum, Proc. Int. Conf. Sendai 1972
K. Kumar, Phys. Rev. Lett. 1972




$$Q^2 \begin{cases} \text{Centroid } \langle S|Q^2|S\rangle \\ \text{Width } \sigma(Q^2) = \sqrt{\langle Q^4\rangle - (\langle Q^2\rangle)^2} \end{cases}$$

$$\cos 3\delta \begin{cases} \text{Centroid } \langle S|Q^3 \cos 3\delta|S\rangle \\ \text{Width } \sigma(\cos 3\delta) = \sqrt{\frac{\langle Q^6 \cos^2 3\delta\rangle}{\langle Q^6\rangle} - \left(\frac{\langle Q^3 \cos 3\delta\rangle}{\langle Q^3\rangle}\right)^2} \end{cases}$$

- Identities relate the 4 invariants for cos3δ, 3 for $\sigma(Q^2)$, and 5 for σ(cos3δ) and provide a test of completeness and convergence.

- Invariants are ideal for a low-precision, model-independent analysis for shape-transitional nuclei.

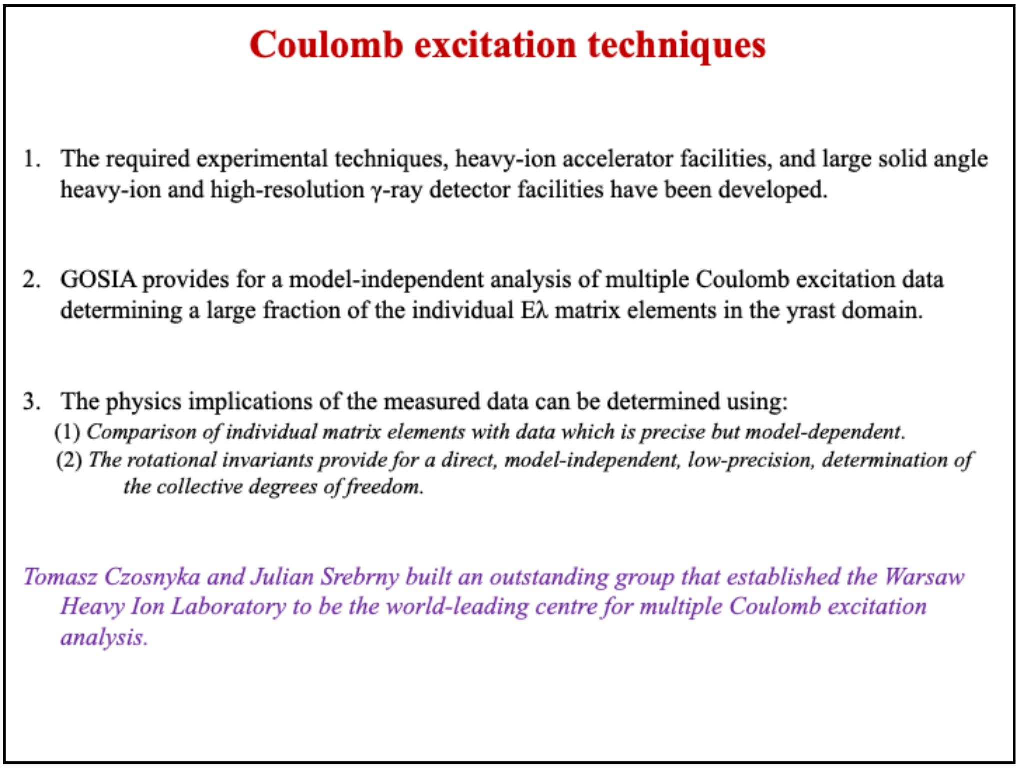
The rotational invariants can be used to determine the centroids, variance, etc of the E2 distribution in the intrinsic frame for each state. For a collective band these centroids and variances for the different states should be correlated.

Assumption of a geometrical collective model allows extraction of the model-dependent shape parameters (β,γ) from the model-independent E2 parameters (Q,δ) if so desired.

While the calculation of the rotational invariants is straightforward, the estimation of their errors must take into account the correlation pattern of the matrix elements involved in the calculation. In many cases the products of the matrix elements that define the invariants can be determined by experimental Coulomb excitation data more accurately than the individual matrix elements themselves, therefore it is not possible to use the errors ascribed to the individual matrix elements to evaluate the errors for the invariants without taking into account the strong cross-correlations between the matrix elements. The only viable method to include the correlation of the matrix elements in evaluation of the error for each rotational invariants is to construct an ellipsoidal error contour in space of the matrix elements containing the estimated confidence region as determined by the error calculation discussed previously.

The quadrupole rotational invariants code SIGMA, which is a part of the GOSIA suite of computer codes, uses the information compiled by GOSIA during error estimation to parameterize the error contours for the invariants. SIGMA calculates the zero-coupled products plus their errors for invariants up to sixth order which allows not only determination of the collective parameters, but also the verification of the consistency of the data analysis as a whole.

The rotational invariant method directly and model independently projects quadrupole collectivity out of Coulomb excitation data. This method is especially powerful for collective-model intepretation of data for shape transitional nuclei. It is a powerful way of projecting a low-precision, but model-independent determination of the essential collective degrees of freedom. For strongly deformed nuclei the comparison of individual matrix elements with calculated values is the more sensitive approach but for shape transitional nuclei the rotational invariants are superior. The power of the rotational invariant method was a strong driver in our exploitation of multiple Coulomb excitation.




The prior discussion shows that:

- The required heavy-ion accelerator facilities now are available.

- The required experimental detector facilities have been developed.

- GOSIA allows for a model-independent least-squares fit of the Eλ matrix elements to Coulomb excitation data.

- The physics implications can be derived two ways.

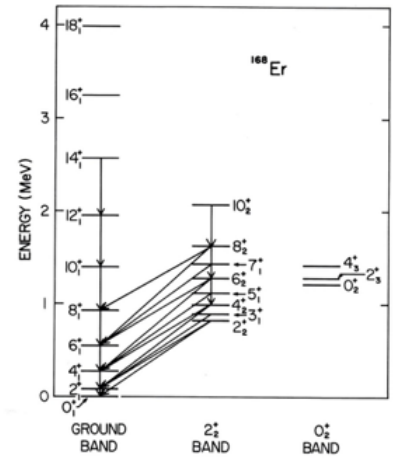
a) Comparison of each individual matrix elements with corresponding model-dependent predictions is precise but the implications can be ambiguous.

b) The rotational invariant technique provides a model-independent, low-precision determination of the collective shape degrees of freedom.

- The pivotal role of the University of Warsaw in the above developments established the heavy-Ion Laboratory in Warsaw to be a world leading centre for multiple Coulomb excitation analyses.

## Strongly deformed nucleus $^{168}$Er

- Strongly-deformed nucleus with 36 known rotational bands.
- $B(E2; 0^+_1 \rightarrow 2^+_1) = 213$ single-particle units.
- Studied the ground and gamma bands
- Measured 900 γ-ray yields using $^{40}$Ca, $^{58}$Ni, and $^{208}$Pb beams.
- Recoil-distance lifetime measurements for 8 states.
- Determined relative signs and magnitudes for 50 E2 matrix elements.

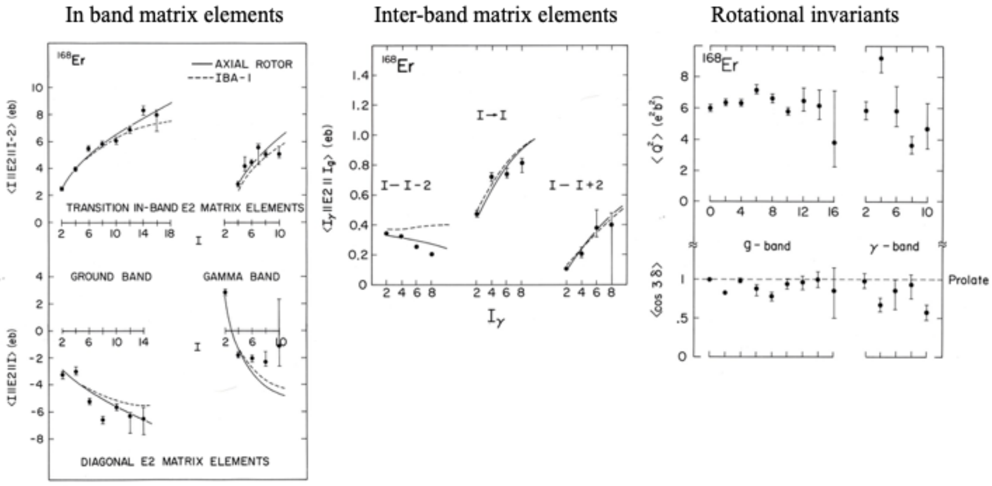



B. Kotlinski et al. Ph.D. thesis, University of Rochester, 1984; Nuclear Physics A517 (1990) 365

$^{168}$Er is a well-studied strongly-deformed nucleus and thus it was the focus of one of the first applications of the multiple Coulomb excitation techniques since this nucleus is known to behave like a prolate deformed rotor.

Danek Kotlinski studied this nucleus via multiple Coulex using the techniques described in the slide.

## E2 matrix elements for $^{168}Er$




**Conclusions:**

The level energies, the transition and diagonal E2 matrix elements and the rotational invariants for the ground and gamma-band all are consistent with rotation of quadrupole rigid rotor with $\delta = 8^0$.

(B. Kotlinski et al. Ph.D. thesis, University of Rochester, 1984; Nuclear Physics A517 (1990) 365)

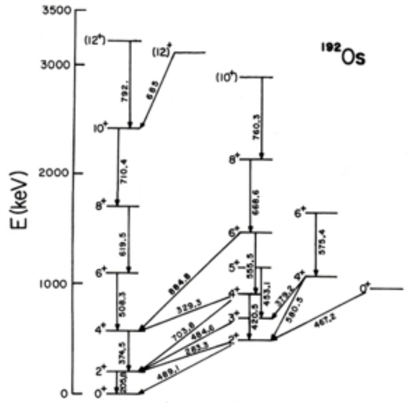
The transition and diagonal E2 matrix element for the ground and gamma band obey the spheroidal rotor relation assuming a prolate deformed shape.

The interband E2 matrix elements are consistent with an average triaxiality of gamma=8$^o$.

The Interacting boson model does not agree with the data.

The rotational invariants lead to the same conclusions as derived from comparing the individual matrix elements with the model predictions. .

**Prolate to oblate shape transition in $^{186,188,190,192}$Os, $^{194}$Pt**




- The energy level diagrams exhibit a band structure characteristic of triaxial quadrupole rotation.
- $B(E2;0_1^+\text{-}2_1^+)$ values range from 90 to 40 Weisskopf single-particle units.
- Do the E2 properties obey a triaxial quadrupole rotor relation?
- Does the quadrupole motion correspond to γ-rigid or γ-soft triaxial collective motion?

(C. Y. Wu; Ph.D. thesis, University of Rochester (1983), C.Y. Wu et al, Nucl. Phys. A607 (1996) 178)

Wu studied the prolate to oblate shape transition in the osmium-platinum nuclei.

The level spectra of these nuclei are characteristic of triaxial deformed rotor as illustrated for $^{192}$Os

The level spectra contain one γ-phonon $2^+$ band plus $0^+$ and $4^+$ double γ-bands

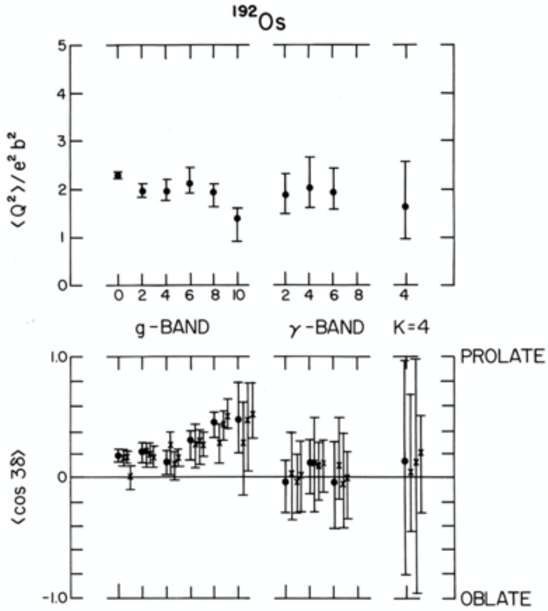
The open questions were:

Do the E2 properties for this nucleus obey a triaxial rotor collective model?

What is the average triaxiality?

Is the collective motion γ rigid or γ soft?

**Rotational invariants for $^{192}Os$**

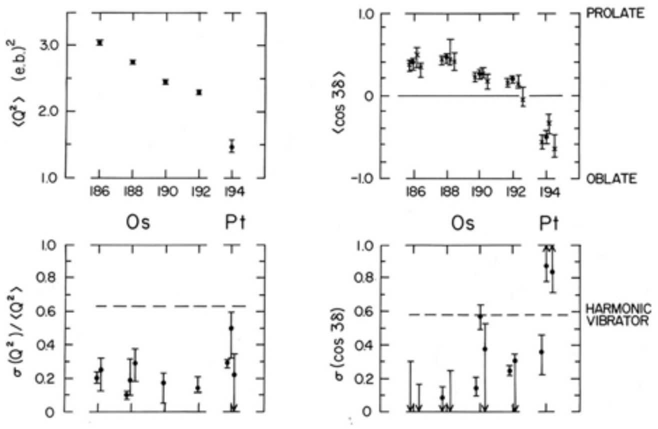



- The magnitude $\langle Q^2 \rangle$ is roughly constant.
- The centroid of the triaxiality is about $26^0$

C. Y. Wu; Ph.D. thesis, University of Rochester (1983),
C.Y. Wu et al, Nucl. Phys. A607 (1996) 178

For $Os^{192}$ the spin dependence of the rotational invariants correspond to rotation of a triaxial rotor with fairly constant deformation having an average triaxiality of $\delta=26^0$.

## Ground state E2 invariants for $^{186,188,190,192}Os$, $^{194}Pt$




- The Coulomb excitation data support a prolate to oblate shape transition on these nuclei.
- The collective motion is soft to γ vibrations of the triaxiality of the collective motion.
- Microscopic models reproduce the general trends but not the fine details of the E2 properties.

C. Y. Wu; Ph.D. thesis, University of Rochester (1983),
C.Y. Wu et al, Nucl. Phys. A607 (1996) 178

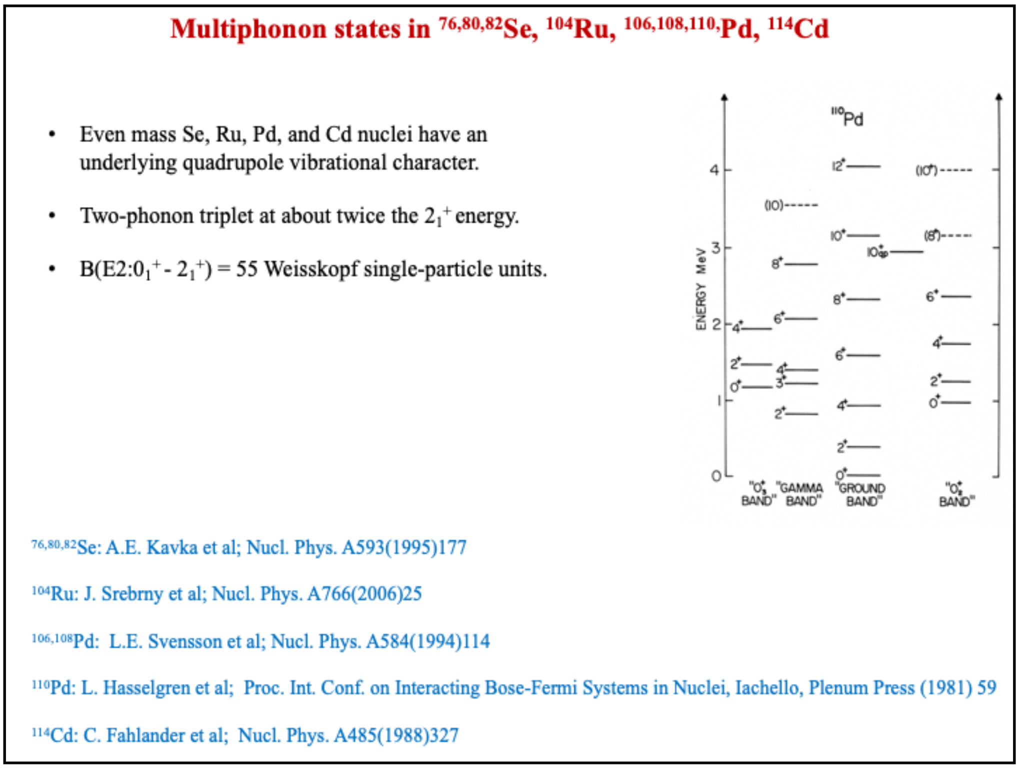
The mass dependence of the ground-state rotational invariants shows:

The gradual drop off in E2 collectivity as one approaches the $^{208}Pb$ closed shell.

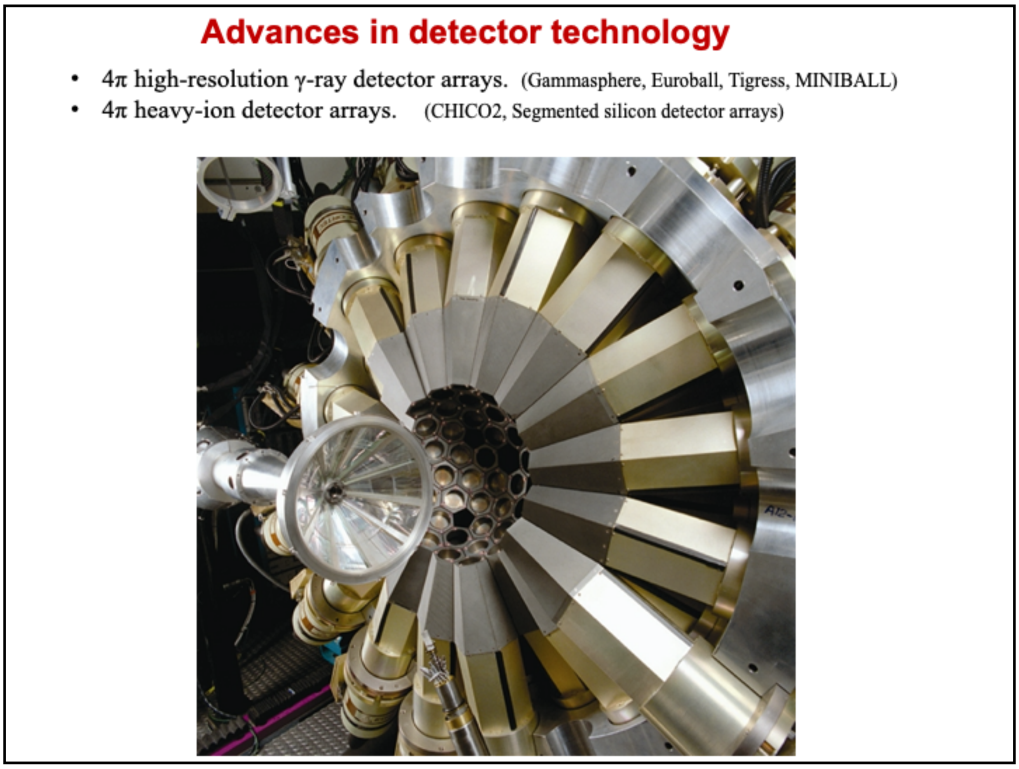
A smooth prolate to oblate shape transition occurs.

The softness to the magnitude $Q^2$ is relatively small.

There is strong evidence for a substantial increase in the gamma softness for $^{194}Pt$ which implies that these nuclei have appreciable gamma softness.




- For many years the even mass Ru, Pd, and Cd were viewed as having an underlying quadrupole vibrational character.
- For example, the level spectrum for $^{110}$Pd has a one phonon 2+ state, a two-phonon 0+,2+,4+ triplet at twice the energy, and a three phonon 0,2,3,4,6 quintuplet at three times the energy.
- The E2 collectivity in $^{110}$Pd is 55 Weisskopf single-particle units.
- An intruder band was observed in $^{110}$Pd which is 30% more deformed than the basic phonon system. Intruder bands were observed in other nuclei such a $^{114}$Cd
- These shape-transitional nuclei exhibit complicated quadrupole phonon structure including intruder bands which is consistent with the predictions of some microscopic models of nuclear structure.




The earliest studies, that have been mentioned, involved use of 1980's technology. More recently there have been two tremendous technical advances that have revolutionized the field.

The first is in detector technology.

Gammasphere is a prime example of modern day gamma-ray detector arrays.

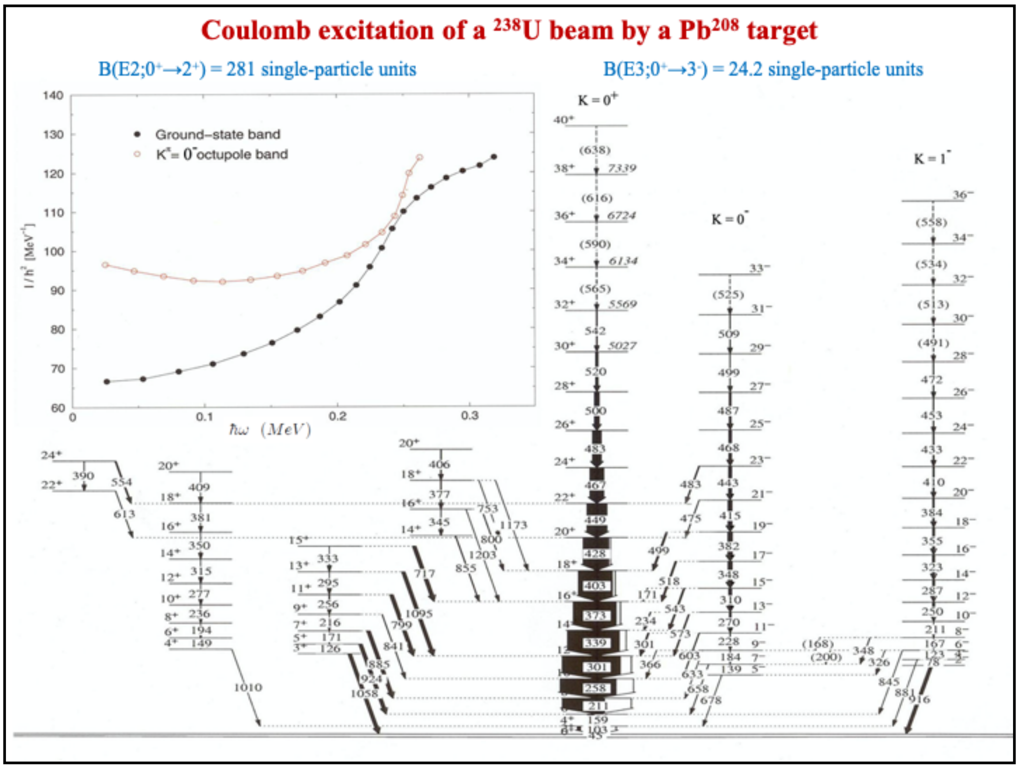
It comprises 110 Compton-suppressed Ge detectors. Photo shows one hemisphere with 55 detectors.

I was heavily involved in development of the Gammasphere in the 1980's and early 1990s.

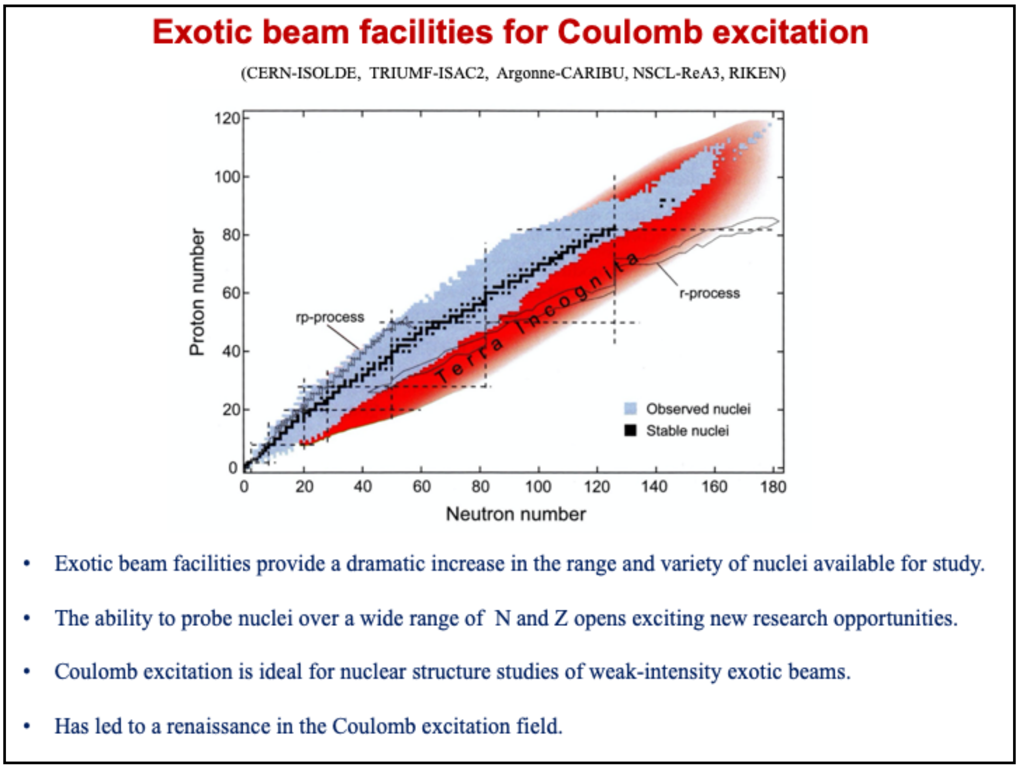
At Rochester we developed the CHICO heavy-ion detector array of parallel-plate gas avalanche detectors specifically for use with Gammasphere. The photo shows 10 individual PPAC's in a conical array in the front half of Chico plus half of Gammasphere.

CHICO provides 1 degree angle resolution in theta and phi plus 500ps time resolution which corresponds to about 5% mass resolution.

In many cases arrays of segmented silicon detectors have been used for heavy-ion detection with radioactive beams.




- The dramatic improvement in the performance provided by Gammasphere plus CHICO has led to an order of magnitude greater resolving power and efficiency.
- An example is Coulomb excitation of a $U^{238}$ beam by a $Pb^{208}$ target.
- $^{238}U$ exhibits the properties of the classic prolate-deformed rotor.
- Almost a 50% increase in spin states of the observed states. That is, we extended observed states in the ground band spin from mid 20's to 40. Also extended the octupole collective bands to spin 36-.
- The insert shows that the moment of inertia increases with angular velocity as the microscopic structure evolves with increasing spin due to the Coriolis force.
- The negative parity bands are excited by E3 Coulex.
- K=0+ and K=0- collective bands are observed to high spin.
- The quadrupole collectivity gives B(E2: 0 – 2+) = 281WU
- The octupole collectivity gives B(E3: 0 – 3-) = 24.2WU
- This octupole collectivity corresponds to octupole vibrations of a rotating strongly-deformed prolate nucleus.
- Note that Coulomb excitation is the ONLY direct probe of octupole collectivity.




- The second revolution that has occurred is the development of exotic beam facilities.
- The chart of the nuclide landscape shows the wide range of nuclei that lie between the neutron and proton drip lines. Nuclei already accessed via nuclear reactions are shown in gray, while the red region could become accessible using future exotic beam accelerator facilities.
- Coulomb excitation requires the nucleus of interest to be available as either a target of beam which has effectively limited Coulomb excitation studies to stable nuclei.
- Stable nuclei shown in black, comprise a small fraction of the nuclear landscape.
- Exotic beam facilities have provided a dramatic increase in the range of nuclei available for study by allowing study of exotic nuclei
- Coulomb excitation has many advantages for study of exotic nuclei at beam intensities that can be $10^7$ times weaker than typical for stable heavy-ion beams.
- The Coulomb excitation cross sections are large, the experiments are clean, and the physics interpretation is straightforward.
- The dashed lines denote proton and neutron shell closures for stable nuclei. These shell closures are quenched when you move away from the valley of stability.
- The ability to probe nuclei over a wide range of N and Z opens exciting new research opportunities for probing the isospin dependence of nuclear structure.
- Exotic beam facilities have stimulated a tremendous resurgence and renaissance in the field of Coulomb excitation.

## Recent exotic-beam studies of multiple Coulomb excitation

**TRIUMF-ISAC2:**

| | |
|---|---|
| **$^{11}$Be** | Weakly-bound neutron-halo nucleus. Inversion of $1p_{1/2}$ $2s_{1/2}$ levels. Measured the strongest known B(E1) and compared with *ab-initio* shell model calculations. (Kwan et al, Phys. Letts B(2013) Submitted ) |
| **$^{20,21,29}$Na, $^{21}$Ne** | Shell gaps and structure near the proton and neutron driplines. Astrophysics. (Schumaker et al, Phys. Rev. C80(2009) 044325, Hurst et al, Phys. Rev. C81(2010) 014312 ) |

**CERN-ISOLDE:**

| | |
|---|---|
| **$^{92,94,96}$Kr** | Shape coexistence and isospin-dependent phase transitions. (Albers et al, Phys. Rev. Letts. 108 (2012) 062701, Albers et al, Nucl. Phys. A899(2013)1) |
| **$^{220}$Rn, $^{224}$Ra** | Octupole collectivity and reflection asymmetry. (Gaffney et al, Nature 497(2013) 199) |

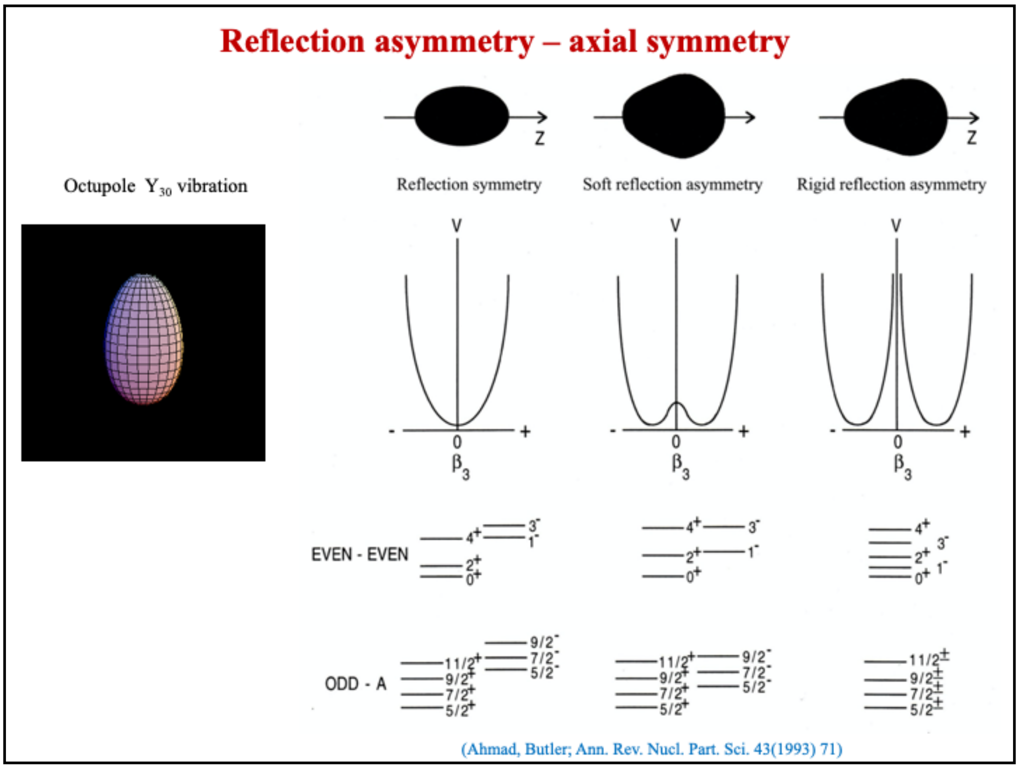
This slide lists examples of our recent studies that we have made that used exotic beam facilities.

$^{11}$Be: Measured the strongest known $B(E1;1/2^{-} \rightarrow 1/2^{+}) = 0.102(2)e^{2}fm^{2}$ to compare with *ab-initio* shell model calculations in this weakly-bound neutron-halo nucleus.

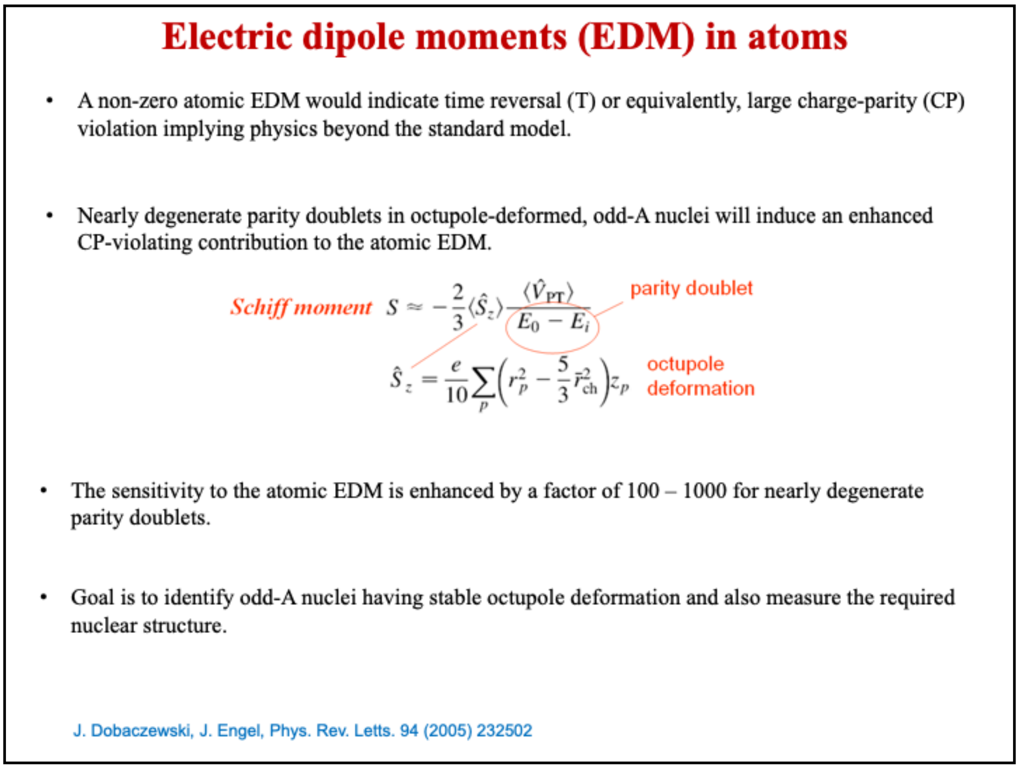
$^{20,21,29}$Na: Studied evolution of shell gaps as a function of neutron number from near the proton to neutron drip lines. $^{20}$Na is of interest for delineating the breakout from hot CNO to the rp process. $^{29}$Na shows the narrowing of the sd-fp shell gap.

$^{92,94,96}$Kr : Probed shape coexistence and isospin dependent phase changes.

Wish to discuss in detail our latest result for $^{220}$Rn and $^{224}$Ra published in the 9 May edition of Nature.




- Octupole correlations imply reflection asymmetry in the intrinsic frame.
- The K = $0^-$ vibration of $^{238}$U corresponds to a $Y_{30}$ oscillation of the reflection-symmetric prolate spheroid illustrated by the video.
- An interesting question in nuclear physics; do rigidly deformed reflection asymmetric intrinsic shapes occur in nuclei.
- Manifestation of octupole deformation are parity doublets in odd-A nuclei and alternating parity states in even-A nuclei.
- The existence of parity doublets in odd-A nuclei has an important application to measurement of the electric dipole moment of an atom and possible extensions to the standard model.

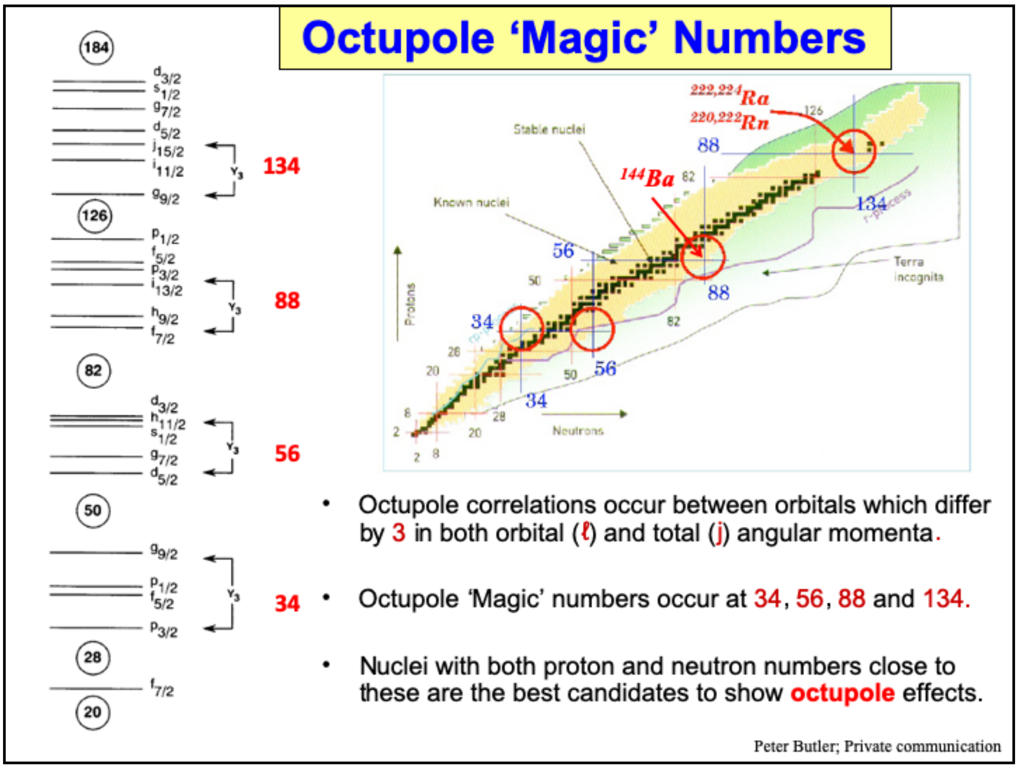



Because of screening by atomic electrons the nuclear Schiff moment, rather than the nuclear EDM, is the quantity that directly induces an atomic EDM.

$V_{PT}$ is the T and P violating nucleon-nucleon interaction mediated by the pion.

The nuclear Schiff moment is greatly enhanced if the parity doublet is nearly degenerate. For example $Ra^{225}$ has a 55keV splitting between the ½+ ground and ½- excited state which can enhance the atomic EDM by a factor of 100 – 1000..

The goal of the experiment was to identify odd-A nuclei having stable octupole deformation that will lead to a nearly degenerate parity doublet and measure the required nuclear structure needed for the interpretation of a measured EDM .




Octupole magic numbers occur when both l and j differ by 3.
Note that all such octupole doubly-magic situations occur in unstable nuclei.
The slide identifies those nuclei likely to have stable octupole deformation.
These magic nuclei are Se68, Se90, Ba144, and Ra222

## Search for reflection asymmetric nuclei

**Coulomb excitation studies in 1990's:**

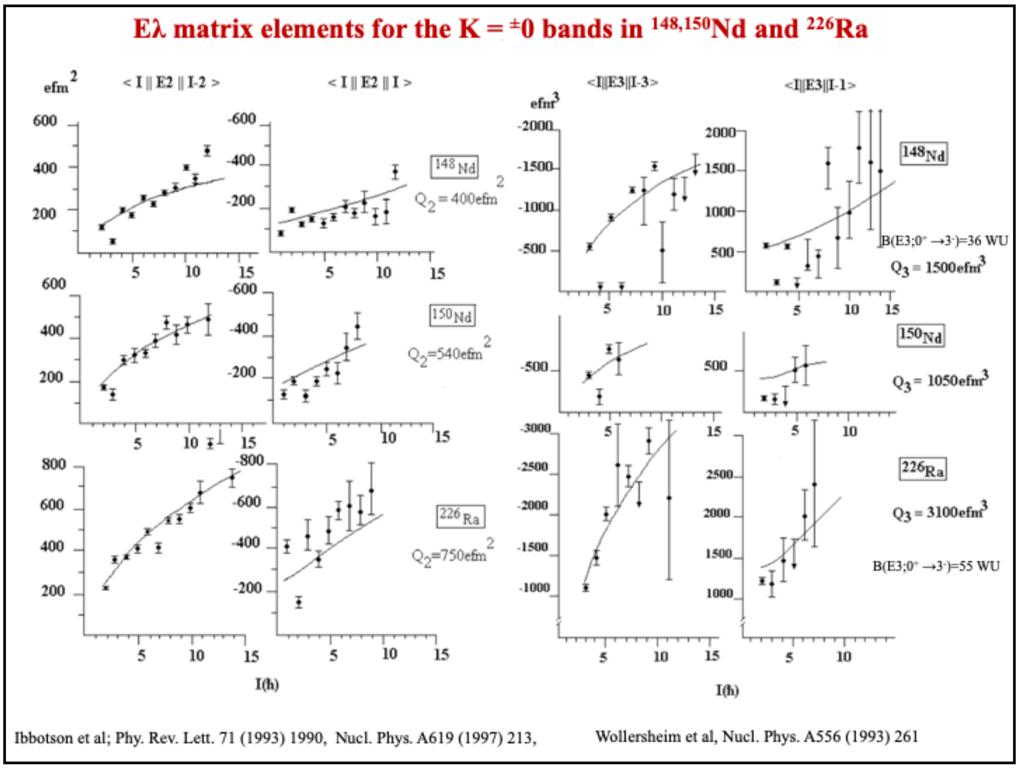
$^{148}$Nd, $^{150}$Nd, $^{150}$Sm: Vibrational octupole motion
Ibbotson et al; Phy. Rev. Lett. 71 (1993) 1990, Nucl. Phys. A619 (1997) 213, Univ. Roch. Ph.D. Thesis 1995.

$^{226}$Ra, Possible stable octupole deformation. Radioactive target (1600yr)
Wollersheim et al, Nucl. Phys. A556 (1993) 261

This slide lists multiple Coulomb excitation studies of octupole deformation that we made during the 1990s .

The Rochester-Liverpool collaboration studied octupole deformation in Nd and Sm isotopes and found that the collectivity corresponded to octupole vibrations of a prolate quadrupole deformed rotor. For $^{148}$Nd the E3 collectivity is about 36Weisskopf single-particle units

The GSI-Warsaw group studied $^{226}$Ra and discovered strong E3 collectivity that is 54 Weisskopf single particle units which suggests stable octupole deformation of a prolate quadrupole deformed rotor.




This slide plots the spin dependence of the measured E2 and E3 matrix elements for $^{148,150}$Nd and $^{226}$Ra measured by multiple Coulomb excitation in the 1990's.

Note that for $^{226}$Ra the measured E2 and E3 matrix elements for both the odd and even spin members of the $K = 0^+$ and $K = 0^-$ bands follow a common rotor prediction which is shown as a solid line which is characteristic for a static quadrupole plus octupole collective deformation. In addition the B(E3:0+ - 3-0= 54 WU indicates strong E3 collectivity.

For $^{148,150}$Nd the measured matrix elements for the even and odd spin states are very different and do not follow the predictions for a rigid rotor. In addition the E3 collectivity of 36WU is weaker than for $^{226}$Ra and thus suggests soft octupole phonon collective motion.

## Search for reflection asymmetric nuclei

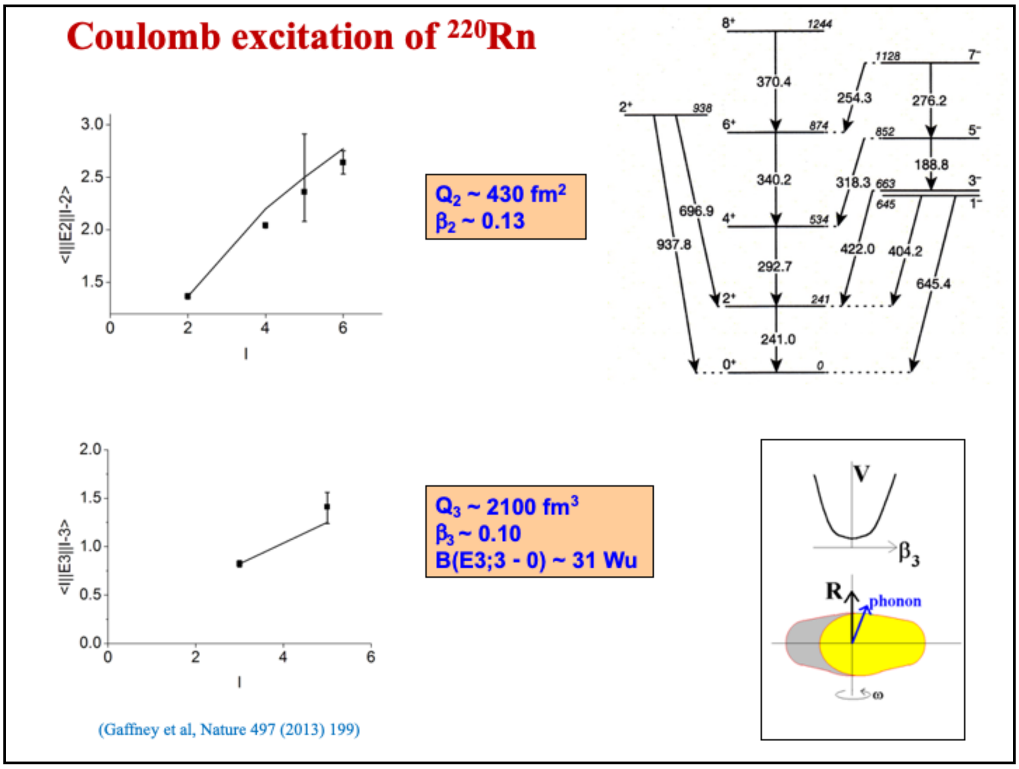
**Coulomb excitation studies 2013:**

| | |
|---|---|
| $^{144}$Ba | ATLAS-CARIBU |
| $^{220}$Rn, $^{224}$Ra | CERN-ISOLDE<br>(Gaffney et al, Nature 497 (2013) 199) |

During 2013 we have been involved in two Coulomb excitation studies of doubly-magic octupole deformed nuclei that lie in two regions where the octupole collectivity is predicted to be a maximum. Both of these studies required exotic beams.

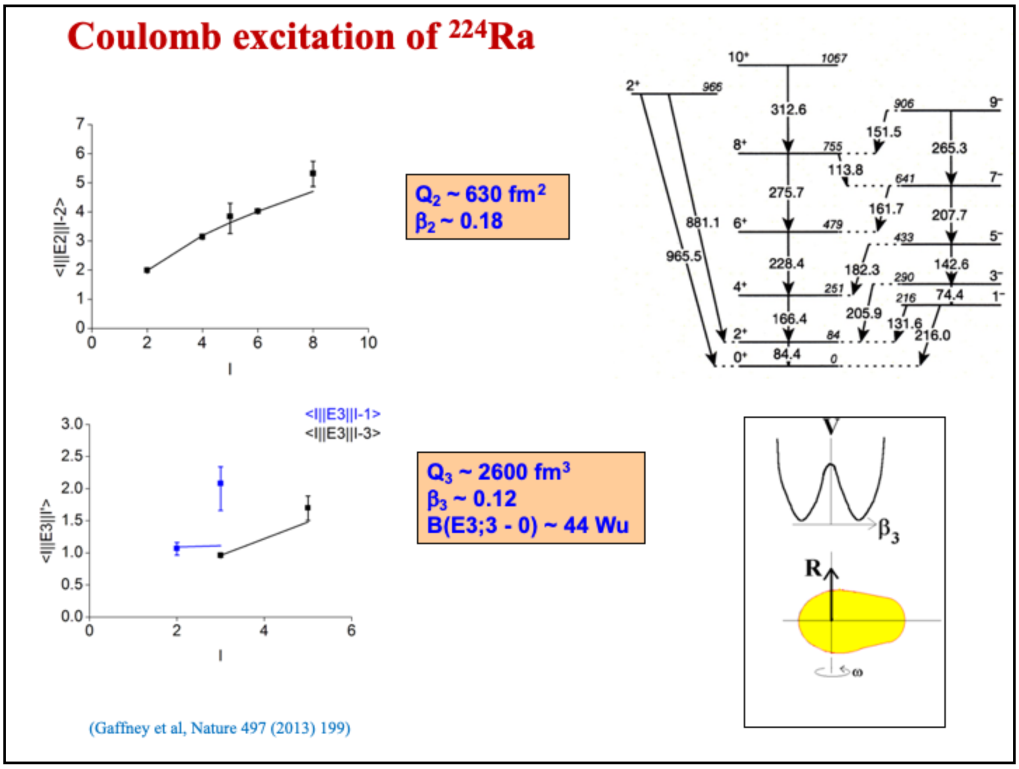
A $^{144}$Ba beam from the ATLAS-CARIBU facility at Argonne was studied using CHICO2 plus Gammasphere. Coulomb excitation of $^{144}$Ba was observed but the beam intensity was insufficient to provide useful E3 matrix elements. This experiment will be repeated later this year using a more intense beam.

Beams of $^{220}$Rn and $^{224}$Ra from the CERN-ISOLDE facility were Coulomb excited successfully and the results have just been published in Nature. The nuclei were produced by spallation in a thick U carbide target bombarded with $10^{13}$ protons/sec at 1.4GeV from the CERN PS Booster. The exotic ions were post-accelerated to 2.83MeV/nucleon by the REX-ISOLDE with a beam intensity of about 3 $10^5$ /s.

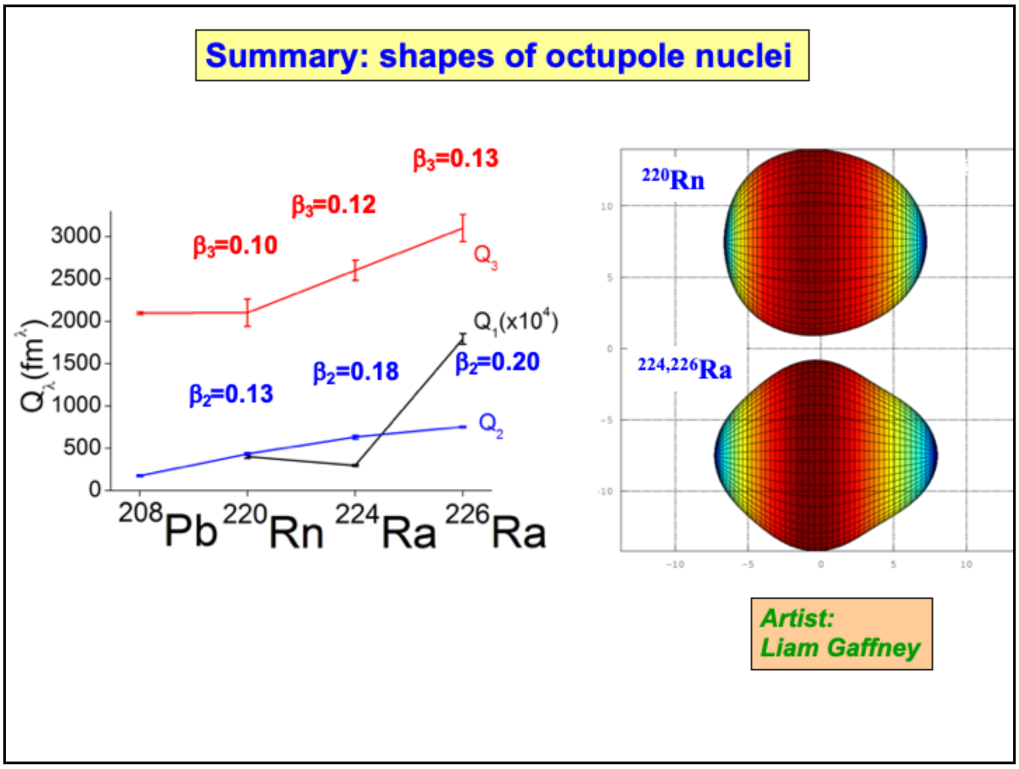



This shows the levels populated plus the E2 and E3 matrix elements.

The measured B(E3) = 31 WU. The Eλ matrix elements and the level structure and E3 strength all suggest soft octupole vibrations of a prolate quadrupole rotor.




The level structure, Eλ matrix elements and the B(E3;3 – 0)=44WU all are consistent with the characteristics of rotation of a prolate quadrupole rotor that has an appreciable static octupole deformation.




This slide summarizes the evolution of the E2, E3 and E1 collectivity with mass above the $^{208}$Pb shell closure. The significantly larger E3 moment for $^{224,226}$Ra suggests an intrinsic shape that has a static axially-symmetric octupole plus prolate quadrupole deformed rotor. By contrast for $^{220}$Rn the prolate quadrupole rotor appears to undergo axially-symmetric octupole oscillations about the prolate shape. Artist impressions of the shape in the intrinsic frame are illustrated. Whereas $^{220}$Rn undergoes reflection-symmetric oscillations of the shape illustrated, $^{224,226}$Ra appear to have a static octupole deformation which rotates perpendicular to the symmetry axis.

For a reflection asymmetric rotor the centre of mass and centre of charge are displaced which implies the existence of a collective E1 component. Unfortunately the single-particle structure also contributes an E1 moment which destructively interferes with the collective E1 component. Also the bulk of the E1 strength is focussed in the high-lying giant dipole resonance. These explain the unusual behaviour and weakness of the observed E1 strengths.

## Studies of octupole nuclei

L.P. Gaffney[1], P.A. Butler[1] , M.Scheck[1,2], A. Hayes[7], F. Wenander[9], M. Albers[2], B. Bastin[12], C. Bauer[6], A. Blazhev[2], A. Blazhev[2], S. Bönig[6], N. Bree[4], J. Cederkall[13], T. Chupp[10], D. Cline[7], T.E. Cocolios[9], J. Diriken[4], T. Grahn[5], A. Herzan[5], M. Huyse[4], D. Jenkins[11], D.T. Joss[1], N. Kesteloot[4], M. Kowalczyk[3], Th. Kröll[6], E. Kwan[8], K. Moschner[2], P. Napiorkowski[3], M. Pfeiffer[2], D. Radeck[2], K. Reynders[4], S. Rigby[1], M. Rudigier[2], S. Sambi[4],, M. Seidlitz[2], P. Thoele[2], P. Van Duppen[4], M. von Schmid[6], D. Voulot[9], N. Warr[2],

K. Wrzosek-Lipska[4], C.Y. Wu[8], M. Zielinska[3]

[2]University of **Cologne**, Germany
[6]Technical University **Darmstadt**, Germany
[14]University of **Edinburgh**, UK
[12]**GANIL**, France
[9]CERN-**ISOLDE**
[5]University of **Jyvaskyla**, Finland
[4]University of **Leuven**, Belgium
[8]Lawrence **Livermore** Laboratory, USA
[1]University of **Liverpool**, UK
[13]University of **Lund**, Sweden
[10]University of **Michigan**, USA
[7]University of **Rochester**, USA
[3]University of **Warsaw**, Poland
[11]University of **York**, UK


This work involved 42 researchers from 14 laboratories. The publication featured on the front page of this years May 9 edition of Nature as shown.

There is an incomplete understanding of the nuclear structure properties as well as the theoretical interpretation of octupole collectivity. Further study is important both for nuclear structure and for interpreting EDM measurements.

The high-energy upgrade of ISOLDE, which is currently in progress, will enable exploration of the unknown properties in both even and odd-A adjacent Rn, Ra and Th nuclei. This will better elucidate where octupole correlations are strongest, identify the region of stable octupole deformation and identify the parity doublets in odd-A nuclei. Gogny-HFB calculations predict that Th and U isotopes with N = 134- 136 will exhibit the largest statically-deformed E3 collectivity which needs to be explored experimentally.

## Future nuclear structure physics

**Nuclear Science:**

Physics of nuclei far from stability is a burgeoning frontier of nuclear physics.

**Radioactive beam facilities:**

**Near term:** ISOLDE, CARIBU, ISAC2, GANIL, ReA3, RIKEN

**Long term:** FAIR, FRIB

Orders of magnitude increase in exotic-beam intensities.

**γ-ray tracking detectors arrays:** GRETA, AGATA,

~ 100-fold improvement in selectivity.

**Conclusion:**

**These new facilities will dramatically expand the nuclear physics research opportunities.**

**Coulomb excitation will play a pivotal role in exploration of nuclei far from stability.**

**Poland pioneered the multiple Coulomb excitation technique and has the expertise to continue playing a major role in exploiting these new research opportunities.**

The future of nuclear physics will focus primarily on the physics of nuclei far from stability.

In the near term a major component of this research will involve Coulomb excitation of radioactive ion beams from radioactive beam facilities such as CARIBU, TRIUMF/ISAC2, REX-ISOLDE, and GANIL.

In the longer term, the orders of magnitude increase in exotic beam intensities provided by the major radioactive beam facilities FAIR and FRIB, now under construction, will greatly advance nuclear structure physics. The experimental program at these accelerator facilities will exploit the powerful new $4\pi$ γ-ray tracking detector systems GRETA, and AGATA that will have single γ-ray peak detection efficiencies of 0.55 at 1.3 MeV; a factor of 6 better than Gammasphere, plus ~1 mm position resolution. Use of such detectors in conjunction with $4\pi$ highly-pixelated heavy-ion detectors such as SuperCHICO will improve the Doppler corrected γ-ray resolution by a factor of 3. As a consequence, these new detector systems could lead to a ≈400 fold improvement in sensitivity for typical Coulomb excitation work which will greatly expand the physics reach and opportunities.

Conclusions:

These new facilities will dramatically expand the research opportunies in nuclear physics.

Coulomb excitation will play a prominent and pivotal role in this research.

Poland pioneered the multiple Coulomb excitation technique and has the expertise to continue playing a major role in exploiting these major new facilities in nuclear science.

# Tribute

**Tomasz Czosnyka 1952 -2006**

Tomasz was a brilliant scientist who made pioneering advances that now form the foundation of modern Coulomb excitation. The legacy of his achievements in nuclear science will endure for a long time.

I want to pay special tribute to Tomasz Czosnyka whose pioneering advances form the foundation of modern multiple Coulomb excitation work.

## Acknowledgements

| | |
|---|---|
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| Alexander Kavka | Pawel Napiorkowski* |
| Bodan Kotlinski * | Mike Schumaker |
| Mike Simon | John F. Smith |
| Anders Backlin | Lars Erik Svensson |
| Ehrenfried Vogt | Nigel Warr |
| Nigel Warr | K. Wrzosek-Lipska* |
| | Magda Zielinski* |

* University of Warsaw Heavy Ion Laboratory

This partial list of collaborators includes those who have played a major role in the development of multiple Coulomb excitation and use of GOSIA. The members of the Warsaw heavy Ion laboratory are designated by asterisks.

Four decades ago I thought that multiple Coulomb excitation could be used to extract complete sets of electromagnetic matrix elements, a dream that would allow evaluation of the rotational invariants for many low-lying nuclear states. The goal was to directly determine the collective shape parameters in the intrinsic frame of reference for many states since these would contribute significantly to the understanding of collective shape degrees of freedom in nuclear structure. I want to thank Tomasz Czosnyka, Julian Srebrny, Lennart Hasselgren, and Ching-Yen Wu, who played pioneering roles in this development, plus the many other collaborators listed here, all of whom have contributed to realization of this dream.