The Challenge of Establishing Triaxial Shapes in Nuclei

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Fundamentals of Nuclear Models: Foundational Models
--David J. Rowe and JLW, World Scientific, 2010 [R&W]
+ second volume nearing completion

Nuclear Triaxiality:

Basic Assumptions

- Nuclear shape—ellipsoid, sharp surface (conserved volume)

$$\frac{x^2}{R_1^2} + \frac{y^2}{R_2^2} + \frac{z^2}{R_3^2} = 1$$

- Moments of inertia

$$\mathcal{J}_1^{\text{rig}} = \frac{1}{5} M (R_2^2 + R_3^2) \,, \qquad \text{etc.}$$

$$\mathcal{J}_1^{\text{irr}} = \frac{1}{5} M \frac{(R_2^2 - R_3^2)^2}{R_2^2 + R_3^2} \,, \qquad \text{etc.}$$

- Bohr approximation for irrotational M. of I.

using

$$R(\theta, \varphi) = R_0 \Big(1 + \sum_{\nu} \alpha_{2\nu} Y_{2\nu}^*(\theta, \varphi) + 0(\alpha^2) \Big)$$

$$\bar{\alpha}_{20} = \beta \cos\gamma \,, \quad \bar{\alpha}_{21} = \bar{\alpha}_{2,-1} = 0 \,, \quad \bar{\alpha}_{22} = \bar{\alpha}_{2,-2} = \frac{1}{\sqrt{2}} \beta \sin\gamma$$

then

$$\mathcal{J}_k^{\text{Bohr}} = 4 B \beta^2 \sin^2(\gamma - k 2\pi/3) \,, \quad k = 1, 2, 3$$

where B is the mass parameter for quadrupole vibrations with irrotational flow.

NOTE*: ratios* of components of M. of I. only depend on γ.

Nuclear Triaxiality

The Bohr Collective Hamiltonian

Bohr collective Hamiltonian

$$\hat{H} = -\frac{\hbar^2}{2B}\nabla^2 + \hat{V},$$

V = V(β,γ), and

$$\nabla^2 = \frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} - \frac{\hat{\Lambda}^2}{\beta^2}$$

where

$$\hat{\Lambda}^2 = -\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} + \sum_{k=1}^{3}\frac{\bar{L}_k^2}{4\sin^2(\gamma - 2\pi k/3)}$$

SERIOUS MISCONCEPTION: the denominator that appears in the location expected for a M. of I. comes from the *SO(5) symmetry of the Bohr Hamiltonian kinetic energy*.

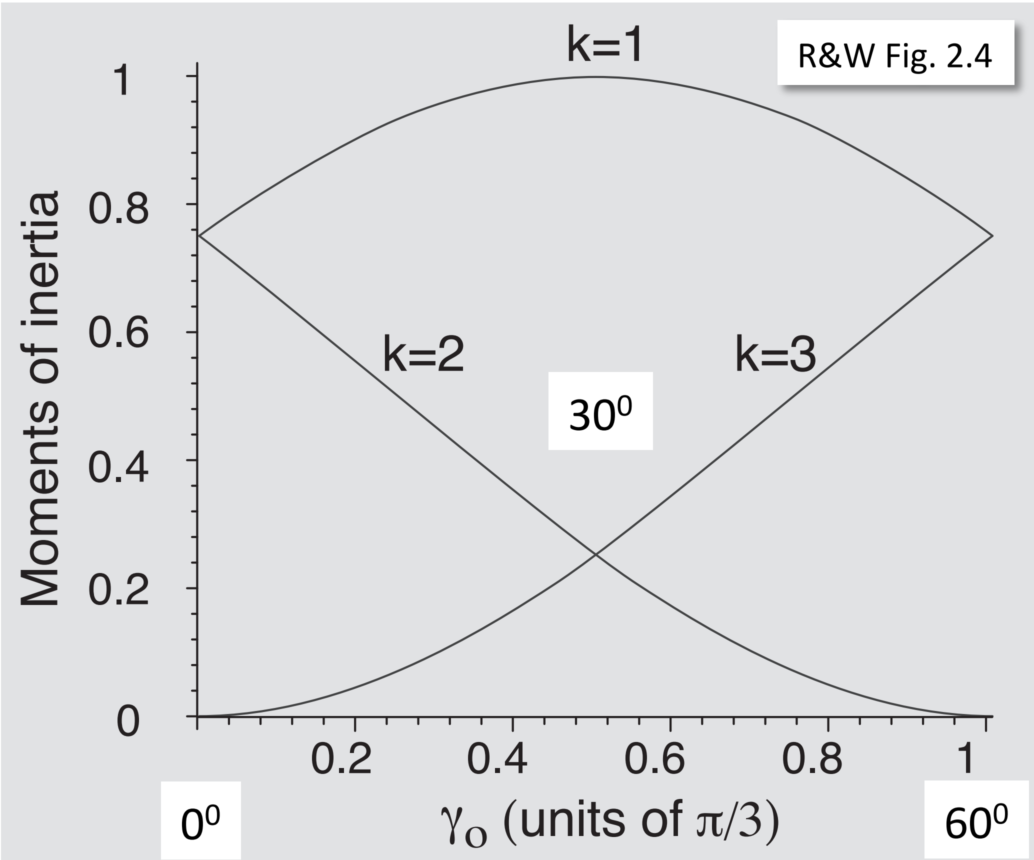
IT DOES NOT PROVE IRROTATIONAL FLOW; BUT, IRROTATIONAL FLOW IS SO(5) INVARIANT.

See D.J. Rowe and J.L. Wood, "Fundamentals of Nuclear Models: Foundational Models", World Scientific, 2010, Chapter 2, p. 106.

The term can be regarded as an SO(5) centrifugal term, cf. SO(3) and $p^2/2m \rightarrow p_r^2/2m + L^2 / 2mr^2$.

Nuclear Triaxiality

peculiarity of the Bohr moments of inertia



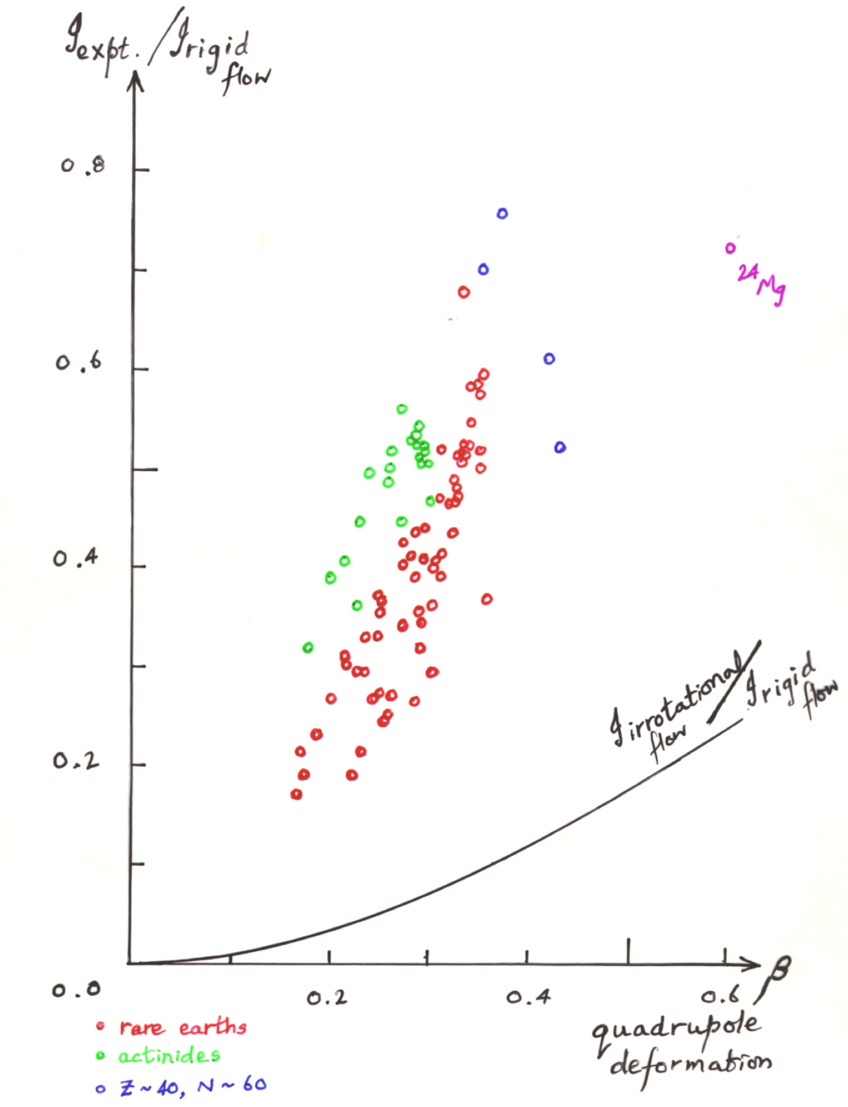
At $\gamma = 30^0$ the inertia tensor is axially symmetric

BUT

the electric quadrupole tensor is not axially symmetric.

Experimental nuclear moments of inertia:

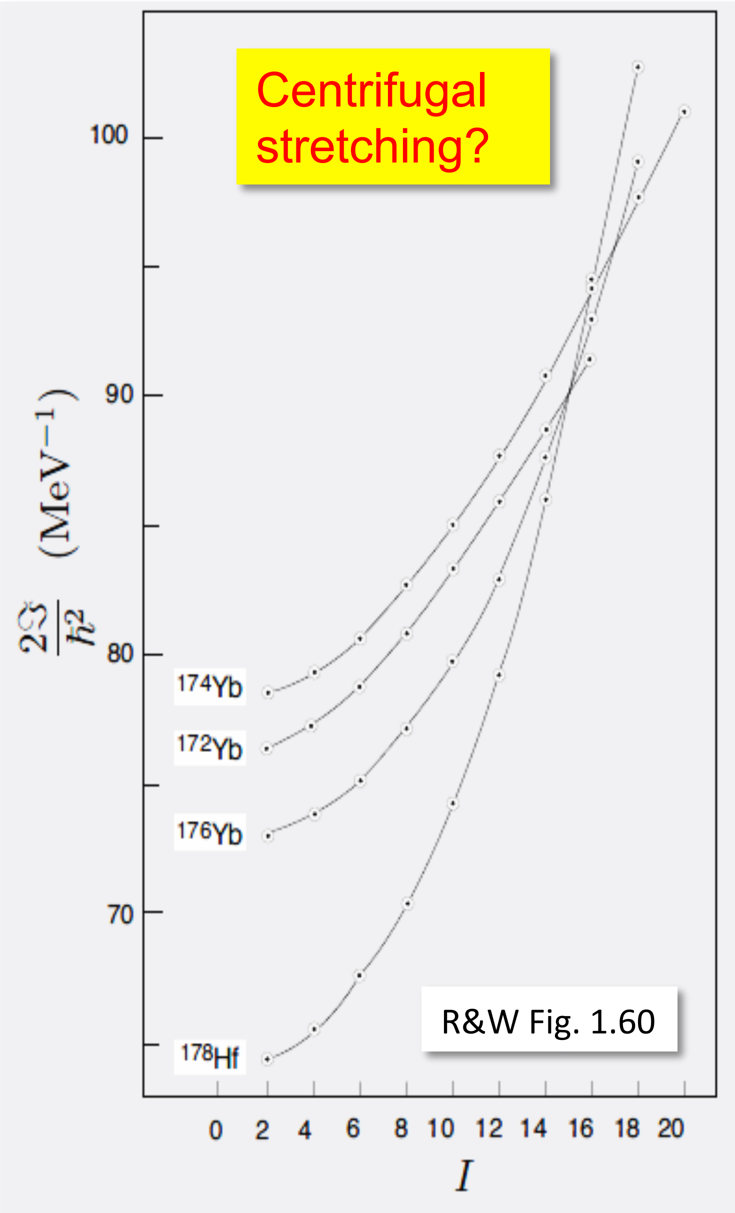
do not reflect either rigid or irrotational flow



These values were obtained by fitting energies to an axially symmetric rotor model.

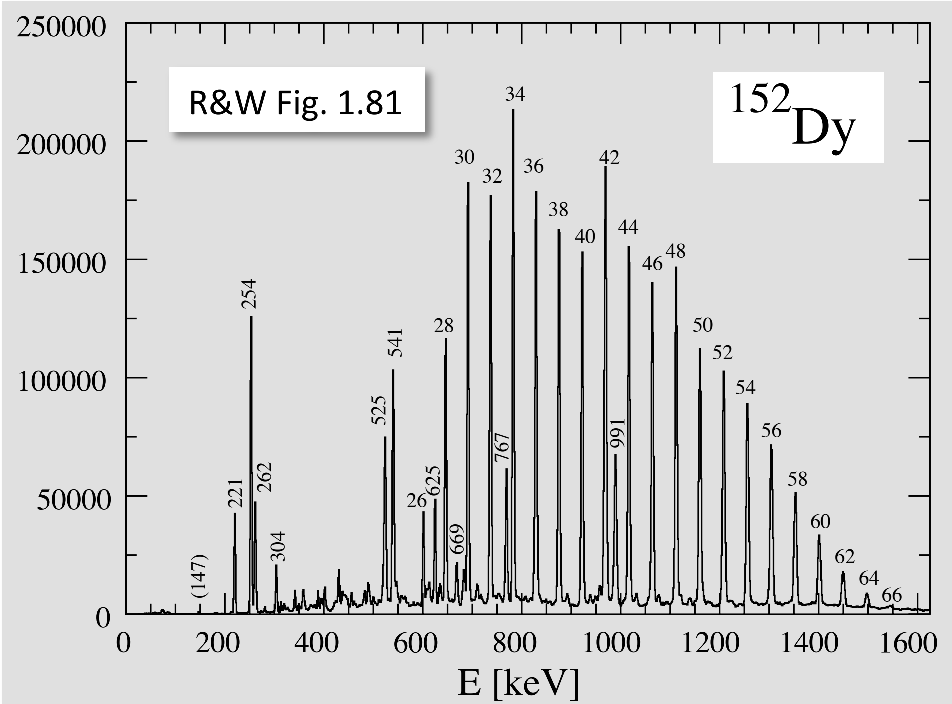
$$\mathcal{J}_k^{\mathrm{Bohr}} \sim \beta^2$$

Rotor Model: axially symmetric
variation of moments of inertia with spin



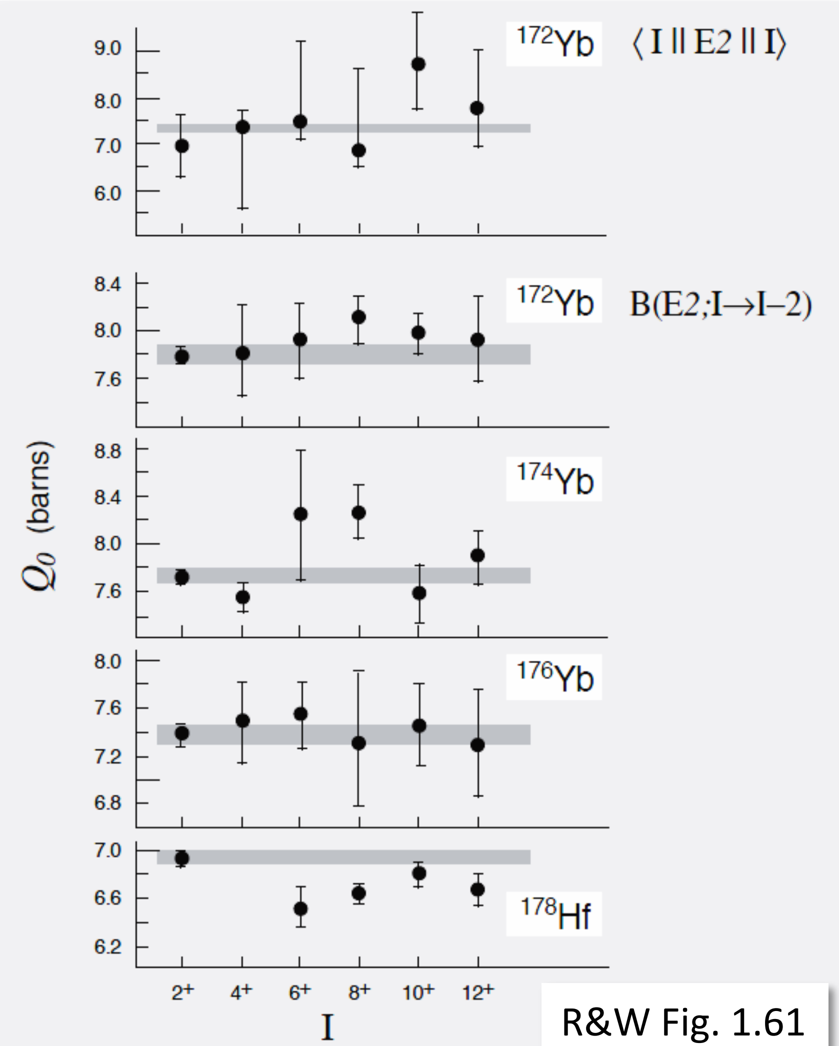
$$E_I = E_0 + \frac{\hbar^2}{2\Im} I(I+1)$$

BUT: superdeformed bands exhibit *zero* centrifugal stretching



Rotor Model:

No variation of intrinsic quadrupole moments with spin: even-mass nuclei, symmetric rotor model parameter fit



R&W Fig. 1.61

No evidence for centrifugal stretching

How do nuclei rotate?

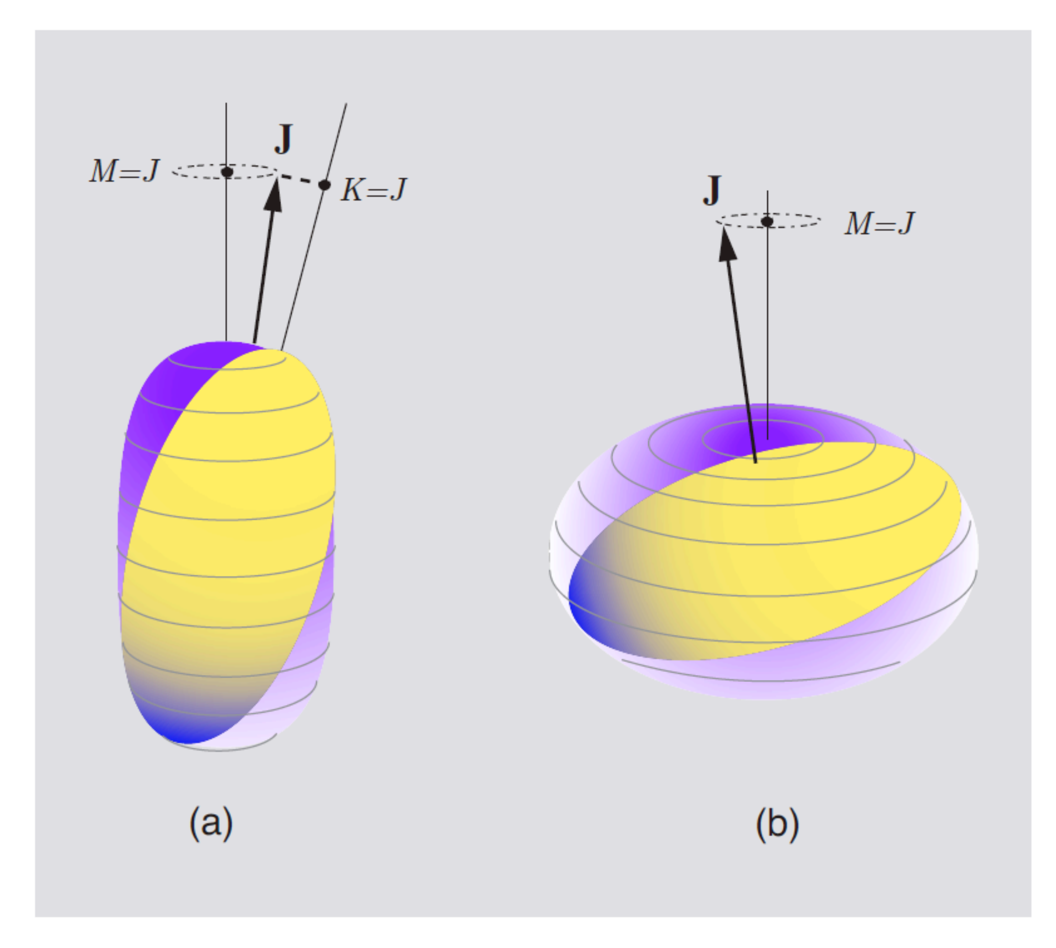
$$B(\mathrm{E2}; \alpha K I_i \to \alpha K I_f) = \frac{5}{16\pi} (I_i K, 20 | I_f K)^2 e^2 |\bar{Q}_0(\alpha K)|^2$$

$$Q(\alpha K I) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} e\bar{Q}_0(\alpha K)$$

rotor model: K qu. no.

NEED MORE, BETTER DATA

Nuclear rotation:
laboratory frame (violet) vs. body frame (yellow)



Prolate rotor: (a) K = J (b) K = 0

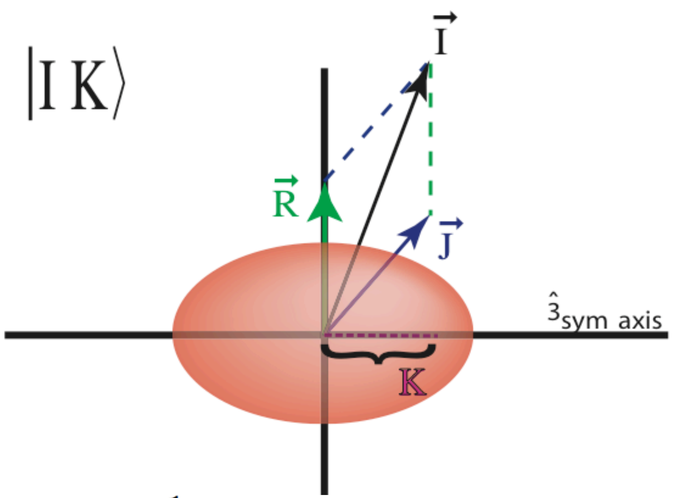
R&W Fig. 1.45

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

Basis—axially symmetric rotor:

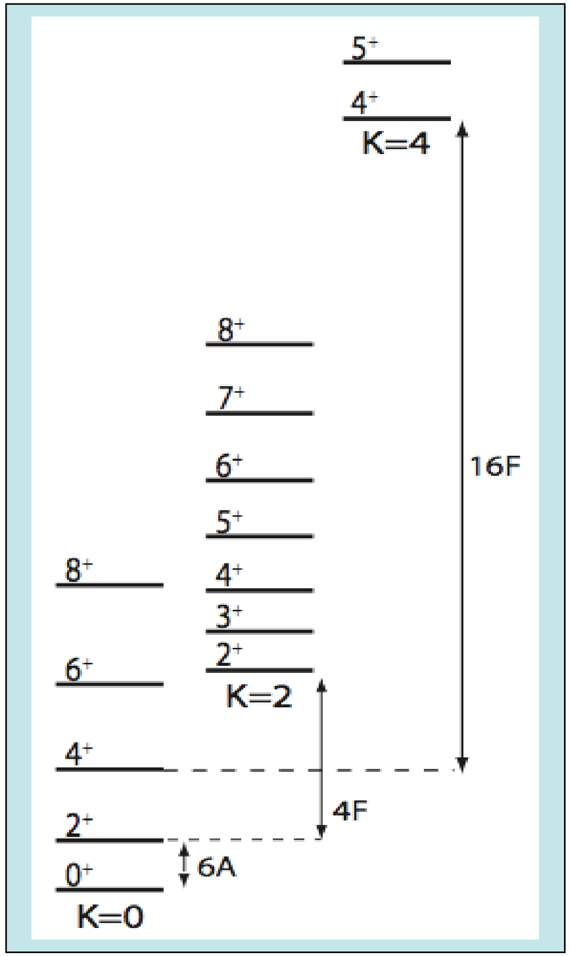
J.L. Wood et al.
PR **C70** 024308 (2004)



$$|IK\rangle_\diamond = \frac{1}{\sqrt{2}}\left(|IK\rangle + (-1)^{I+K}|I,-K\rangle\right)$$

Because nucleus has a plane of reflection symmetry

$${}_\diamond\langle IK|\hat{H}|IK\rangle_\diamond = AI(I+1) + FK^2$$



A triaxial rotor model:

with independent electric quadrupole and inertia tensors

Hamiltonian:

$$\hat{H} = A_1\hat{I}_1^2 + A_2\hat{I}_2^2 + A_3\hat{I}_3^2$$

where $A_1 = 1/(2\mathcal{J}_1)$ and $\mathcal{J}_1$= moment of inertia, etc.

E2 Quadrupole Operator:

$$\hat{T}(E2) = \cos\gamma\hat{T}_0^{(2)} + \frac{\sin\gamma}{\sqrt{2}}(\hat{T}_{+2}^{(2)} + \hat{T}_{-2}^{(2)})$$

where $\langle I_f K_f||\hat{T}_{\pm\nu}^{(2)}||I_i K_i\rangle = Q_0\sqrt{2I_i+1}\langle I_i K_i; 2, \pm\nu|I_f K_f\rangle$

Reduced Transition Probability:

$$B(E2) = \frac{|\langle I_f K_f||\hat{T}(E2)||I_i K_i\rangle|^2}{2I_i+1}$$

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

Hamiltonian: Shape dictated by A_1, A_2, A_3

$$\hat{H} = A_1 \hat{I}_1^2 + A_2 \hat{I}_2^2 + A_3 \hat{I}_3^2 \longrightarrow \hat{H} = A\hat{I}^2 + F\hat{I}_3^2 + G(\hat{I}_+^2 + \hat{I}_-^2)$$

where ${}_\diamond\langle IK|\hat{H}|IK\rangle_\diamond = AI(I+1) + FK^2$ and G produces "mixing"

$\boxed{A = A_1 + A_2}$ $\boxed{F = A_3 - A}$ $\boxed{G = \frac{1}{4}(A_1 - A_2)}$

E2 Quadrupole Operator: Shape dictated by γ

$$\hat{T}(E2) = \cos\gamma \hat{T}_0^{(2)} + \frac{\sin\gamma}{\sqrt{2}}(\hat{T}_{+2}^{(2)} + \hat{T}_{-2}^{(2)})$$

Standard procedure (e.g. Davydov)

A_1, A_2, A_3 are determined by γ through $A_1 = 1/(2\mathcal{J}_1)$, etc.

$$\mathcal{J}_k^{\rm Bohr} = 4B\beta^2 \sin^2(\gamma - k2\pi/3)\,, \quad k = 1,2,3$$

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

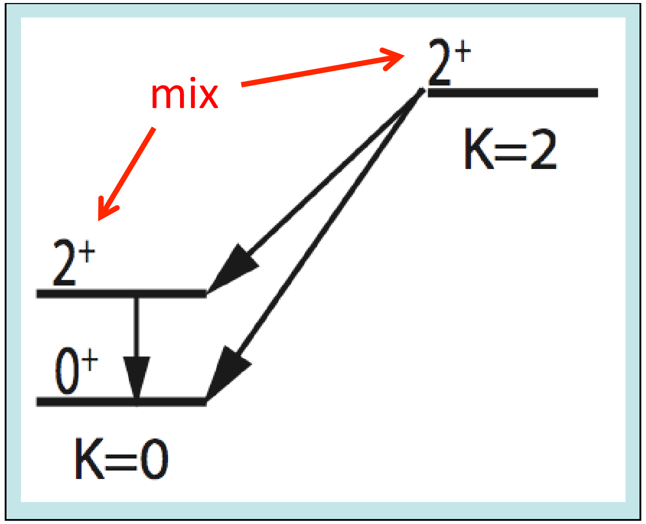
- The spin-0 / spin-2 subspace

$$\langle IMK \pm 2|H|IMK\rangle$$
$$= G\sqrt{(I \mp K)(I \pm K + 1)(I \mp K - 1)(I \pm K + 2)}$$

$$H(2) = \begin{pmatrix} 6A & 4\sqrt{3}G \\ 4\sqrt{3}G & 6A + 4F \end{pmatrix},$$

which yields

$$E(2) = 6A + 2F \pm 2\sqrt{F^2 + 12G^2}.$$



These two equations cannot be solved because they have 3 unknowns.

Extension to the spin-0 / spin-2 / spin-4 subspace encounters deviations due to the spin-dependence of A and F: cannot usefully solve for G.

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

HOWEVER:

$$|2_1^+,M\rangle = \cos\Gamma|2,K=0,M\rangle - \sin\Gamma|2,K=2,M\rangle$$

$$|2_2^+,M\rangle = \sin\Gamma|2,K=0,M\rangle + \cos\Gamma|2,K=2,M\rangle$$

where

$$|I,K=2,M\rangle = \frac{1}{\sqrt{2}}[|I,2,M\rangle + (-1)^I|I,-2,M\rangle]$$

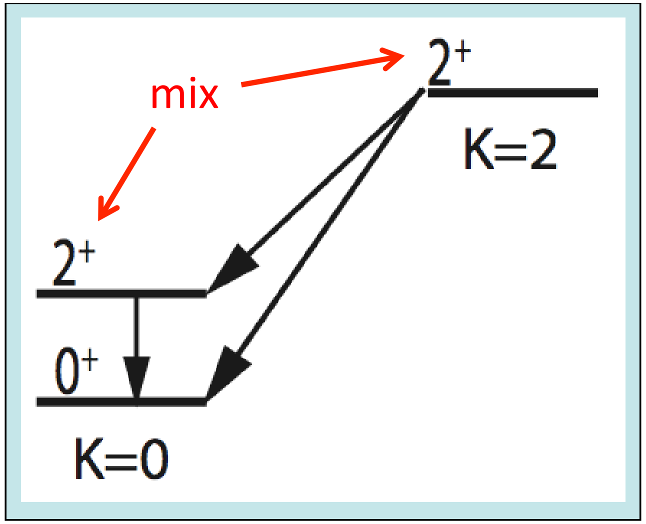
and

$$\tan\Gamma = \frac{\sqrt{F^2+12G^2}-F}{2\sqrt{3}G}.$$

$$\hat{T}(E2) = \sqrt{\frac{5}{16\pi}}\left[\cos\gamma\,\hat{T}_0^{(2)} + \frac{\sin\gamma}{\sqrt{2}}\left(\hat{T}_{+2}^{(2)} + \hat{T}_{-2}^{(2)}\right)\right],$$

where the $\hat{T}_\nu^{(2)}$ reduce to

$$\langle I_f K_f \| \hat{T}_{\pm\nu}^{(2)} \| I_i K_i\rangle = Q_0\sqrt{2I_i+1}\langle I_i K_i; 2, \pm\nu | I_f K_f\rangle$$



UNKNOWNS:

A, F, G or Γ, γ, Q_0

DATA:

$E(2_1^+)$, $E(2_2^+)$ and

B_{20} , $B_{2'2}$, $B_{2'0}$, Q_2 , $Q_{2'}$

7 equations in 5 unknowns

A triaxial rotor model:

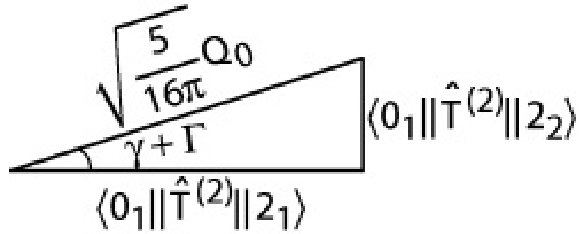
with independent electric quadrupole and inertia tensors

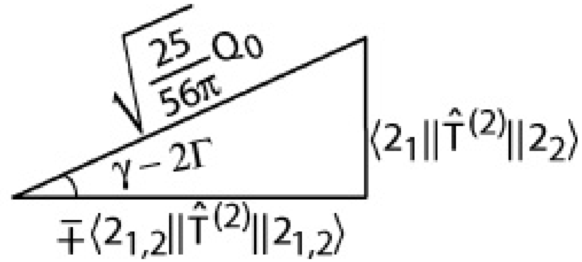
$$\langle 0_1 \| \hat{T}(E2) \| 2_1 \rangle = \sqrt{\frac{5}{16\pi}}\, Q_0 \cos(\gamma + \Gamma),$$

$$\langle 0_1 \| \hat{T}(E2) \| 2_2 \rangle = \sqrt{\frac{5}{16\pi}}\, Q_0 \sin(\gamma + \Gamma),$$

$$\langle 2_1 \| \hat{T}(E2) \| 2_2 \rangle = \sqrt{\frac{25}{56\pi}}\, Q_0 \sin(\gamma - 2\Gamma),$$

$$\begin{aligned}\langle 2_1 \| \hat{T}(E2) \| 2_1 \rangle &= -\sqrt{\frac{25}{56\pi}}\, Q_0 \cos(\gamma - 2\Gamma) \\ &= -\langle 2_2 \| \hat{T}(E2) \| 2_2 \rangle,\end{aligned}$$





$$B(E2; I_i \to I_f) = \frac{\langle I_f \| \hat{T}(E2) \| I_i \rangle^2}{(2I_i + 1)},$$

and quadrupole moments,

$$Q(2_1^+) = -\tfrac{2}{7} Q_0 \cos(\gamma - 2\Gamma) = -Q(2_2^+).$$

7 equations in 5 unknowns:
A, F from $E(2_1^+)$, $E(2_2^+)$;

G or Γ, γ, Q_0 from E2 M.E.'s

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

$$A = \frac{1}{6}E(2_1^+) = 22.86\,\mathrm{keV},$$

$$F = \frac{1}{4}[E(2_2^+) - E(2_1^+)] = 157.6\,\mathrm{keV},$$

$$Q_0 = \sqrt{\frac{16\pi}{5}}\sqrt{\langle 0_1\|\hat{T}(E2)\|2_1\rangle^2 + \langle 0_1\|\hat{T}(E2)\|2_2\rangle^2}$$

$$= 3.171 \sqrt{[(1.674)^2 + (0.545)^2]} = 5.582 \text{ eb}$$

$$\gamma + \Gamma = \tan^{-1}\left(\frac{\langle 0_1\|\hat{T}(E2)\|2_2\rangle}{\langle 0_1\|\hat{T}(E2)\|2_1\rangle}\right) = \tan^{-1}(0.545 / 1.674) = 20.04°$$

$$\gamma - 2\Gamma = \sin^{-1}\left(\sqrt{\frac{56\pi}{25}}\frac{\langle 2_1\|\hat{T}(E2)\|2_2\rangle}{Q_0}\right),$$

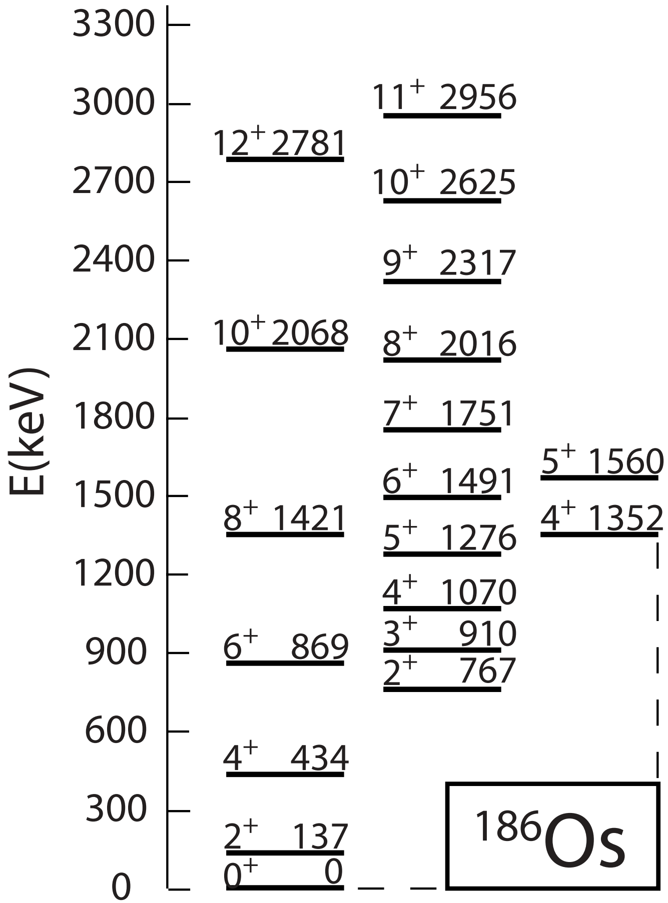
$$= \sin^{-1}(2.653 \times 0.897 / 5.582) = 28.04°$$

whence

$$\Gamma = -\ 2.67°$$
$$\gamma = 22.71°$$

and

$$G = \frac{F}{2\sqrt{3}}\tan(2\Gamma) = -3.82\,\mathrm{keV}.$$

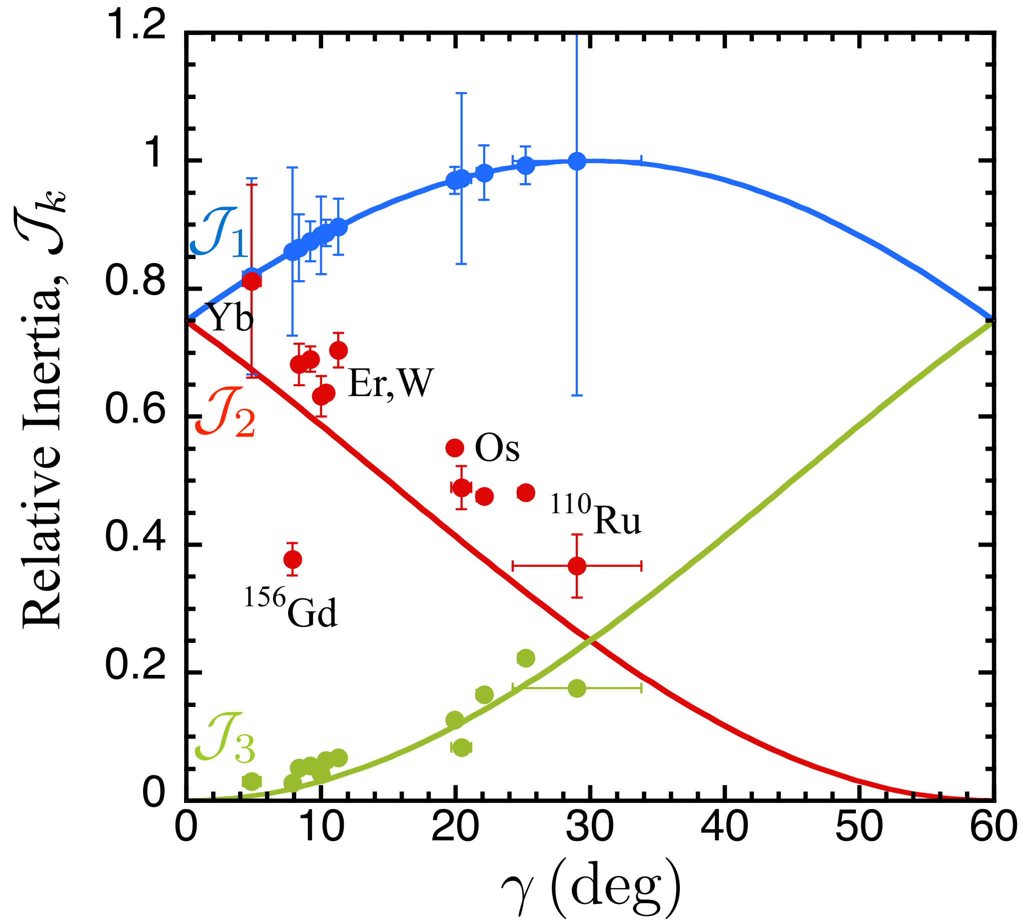


NOTE: 4_3^+ state appears (sub) vibrational

Nuclear moments of inertia fitted to a triaxial rotor model with independent components of the inertia tensor

$$\mathcal{J}_{irrot.,\,k} = 4B_{irrot.}\beta^2 \sin^2\left(\gamma - k\frac{2\pi}{3}\right)$$

J.M. Allmond and JLW
Phys. Lett. **B767** 226 (2017)



k = 1 norm. to data

k = 2

k = 3 symm. axis
@ $\gamma = 0°$ and $60°$

NOTE: relative values do not prove irrotational flow—they conform to the SO(5) invariant form of the inertia tensor.

NEED MORE DATA

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

- Some global perspectives:

$$\Gamma_{\text{irrot}} = -\frac{1}{2}\cos^{-1}\left(\frac{\cos 4\gamma + 2\cos 2\gamma}{\sqrt{9 - 8\sin^2 3\gamma}}\right),$$

For $\gamma = 30°$, $\Gamma_{\text{irrot.}} = -30°$, whence from

$$\langle 0_1 \| \hat{T}(E2) \| 2_2 \rangle = \sqrt{\frac{5}{16\pi}}\, Q_0 \sin(\gamma + \Gamma),$$

$B(E2;\, 2_2^+ \rightarrow 0_1^+) = 0,$

$$\langle 2_1 \| \hat{T}(E2) \| 2_1 \rangle = -\sqrt{\frac{25}{56\pi}}\, Q_0 \cos(\gamma - 2\Gamma)$$
$$= -\langle 2_2 \| \hat{T}(E2) \| 2_2 \rangle,$$

$Q(2_1^+) = 0,$

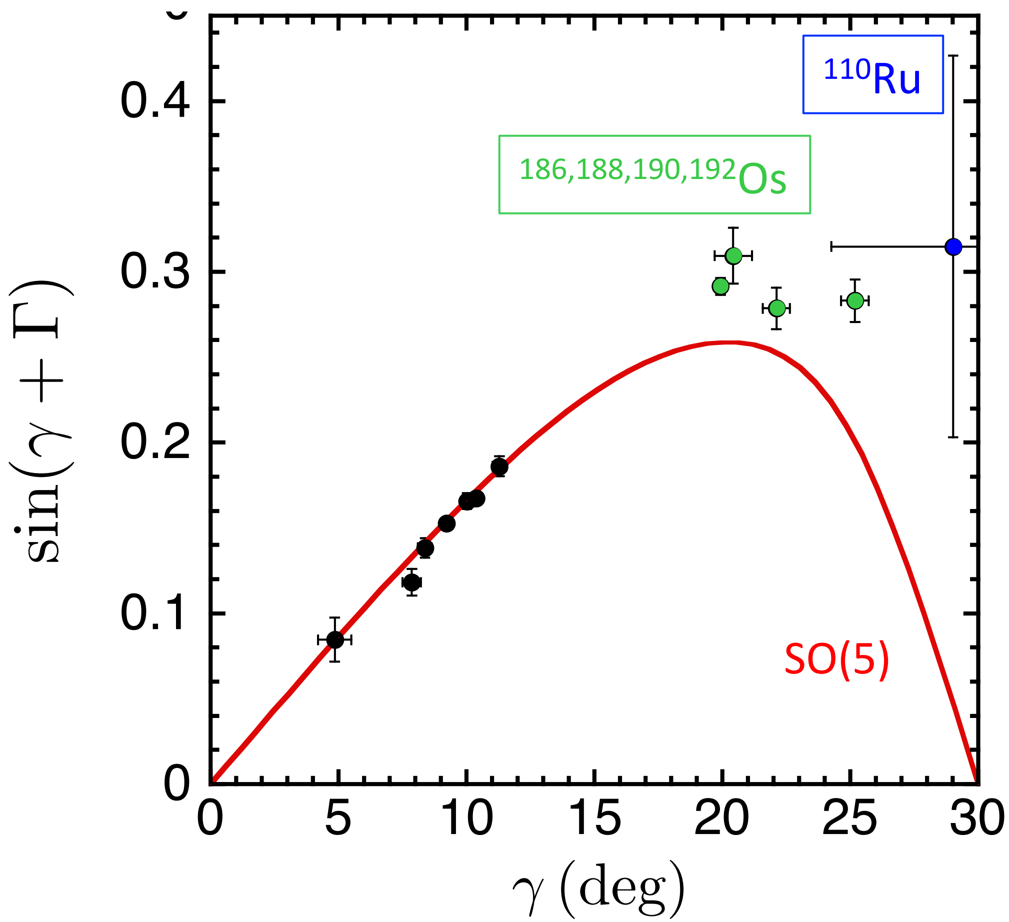
$Q(2_2^+) = 0.$

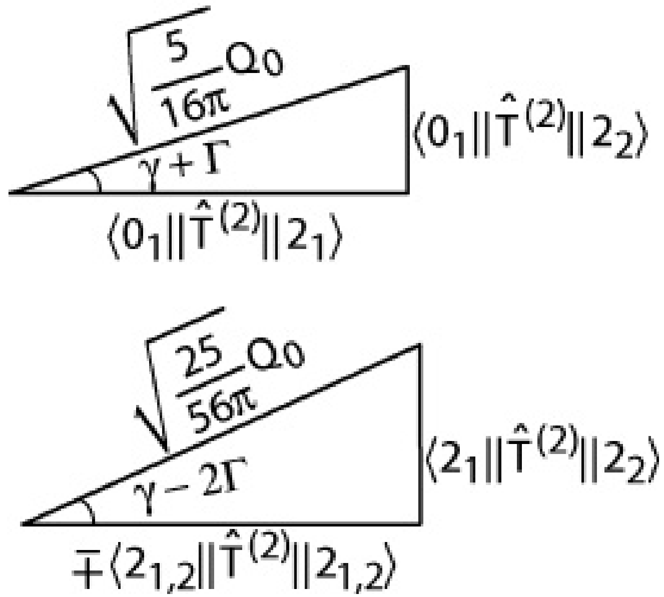
While these properties of the Davydov model at $\gamma = 30°$ were well known;
it was not known that they are due to a destructive interference effect between the electric quadrupole tensor and the inertia tensor.

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

^{110}Ru: D.T. Doherty et al., Phys. Lett. **B766** 334 (2017)

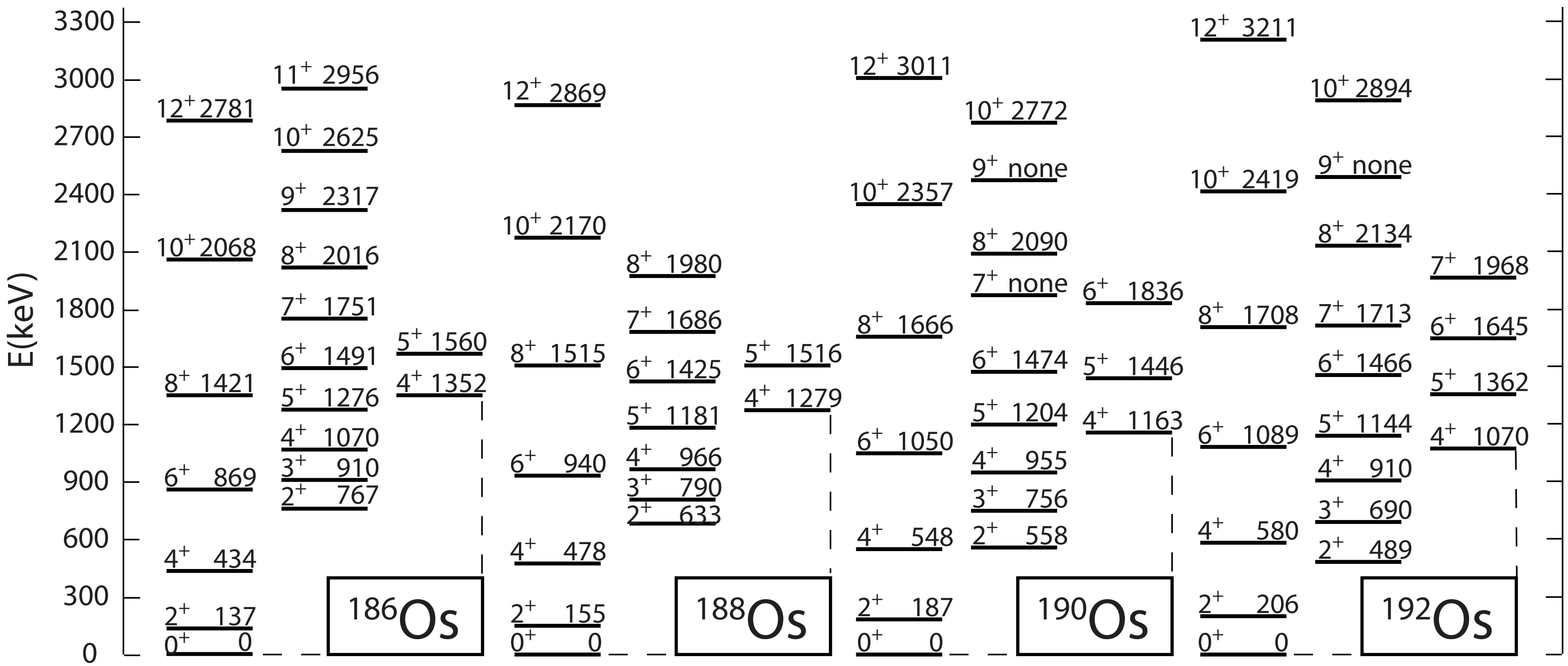




J.M. Allmond and JLW, Phys. Lett. **B767** 226 (2017)

A triaxial rotor model:

case study--the Os isotopes



A triaxial rotor model:

case study--the Os isotopes

<E2> eb calc. (% dev. from expt.)—full diagonalization

	^{186}Os	^{188}Os	^{190}Os	^{192}Os
$2_1 - 0_1$	1.6697 (−0.3%)	1.5812 (−0.2%)	1.5190 (−0.7%)	1.4414 (−1.0%)
$4_1 - 2_1$	2.7170 (−1.6%)	2.5761 (−2.5%)	2.4853 (+5.0%)	2.3628 (+11.7%)
$6_1 - 4_1$	3.512 (−9.7%)	3.338 (+0.8%)	3.2369 (+9.0%)	3.1448 (+7.3%)
$8_1 - 6_1$	4.237 (−1.9%)	4.035 (+1.6%)	4.009 (+7.8%)	3.840 (+7.3%)
$4_2 - 2_2$	1.7509 (−10.9%)	1.661 (−6.7%)	1.6115 (−13.9%)	1.5580 (−4.8%)
$6_2 - 4_2$	2.865 (+3.0%)	2.668 (+8.5%)	2.224 (−14.5%)	2.172 (+3.9%)
$8_2 - 6_2$	3.550 (+8.9%)	3.303 (+29.5%)	3.105 (+19.4%)	2.906 (+25.8%)

Mass	A (keV)	F (keV)	Q_0 (eb)	γ (deg)	Γ (deg)	G (keV)
186	22.86	157.6	5.582	20.43	−2.40	−3.82
188	25.84	119.5	5.254	19.93	−2.98	−3.60
190	31.12	92.8	5.051	22.12	−5.94	−5.64
192	34.30	70.8	4.814	25.19	−8.74	−6.44

J.M. Allmond et al.
PR **C78** 014302 (2008)

Data from:
C.Y. Wu, D. Cline,
T. Czosnyka, et al.
NP **A607** 178 (1996)

A triaxial rotor model:

case study--the Os isotopes

<E2> eb calc. (% dev. from expt.)—full diagonalization Allmond et al.

	^{186}Os	^{188}Os	^{190}Os	^{192}Os
$2_2 - 0_1$	0.5581 (+2.4%)	0.4958 (+2.6%)	0.4800 (+8.1%)	0.4771 (11.0%)
$2_2 - 2_1$	0.8668 (−3.4%)	0.8362 (−3.3%)	0.9888 (−7.2%)	1.1406 (−7.3)
$2_2 - 4_1$	0.2949 (+29.9%)	0.3072 (−18.7%)	0.401 (+111.0%)	0.455 (+30.0%)
$4_2 - 2_1$	0.3471 (−17.2%)	0.2357 (−16.7%)	0.0572,(−71.8%)	−0.0402 (−130.9%)
$4_2 - 4_1$	1.2524 (+2.7%)	1.187 (+7.9%)	1.2849 (−10.5%)	1.309 (−3.1%)
$4_2 - 6_1$	0.634 (−5.4%)	0.640 (+12.2%)	0.867 (+31.3%)	0.587 (+46.8%)
$6_2 - 4_1$	0.1535 (−52.8%)	0.0141 (−88.9%)	−0.3927 (−301.4%)	−0.1797 (−360.4%)
$6_2 - 6_1$	1.406 (+2.6%)	1.276 (−12.6%)	1.123 (−36.2%)	1.105 (−25.9%)
$2_1 - 2_1$	−1.917 (−9.6%)	−1.795 (−3.8%)	−1.627 (−30.2%)	−1.411 (−16.7%)
$4_1 - 4_1$	−2.218 (−9.8%)	−2.017 (−0.8%)	−1.576 (−23.1%)	−1.104 (−51.3%)
$6_1 - 6_1$	−2.261 (−35.40%)	−1.987 (−24.2%)	−1.170 (−28.6%)	−0.822 (+29.2%)
$8_1 - 8_1$	−2.160 (+4.4%)	−1.874 (−35.8%)	−1.234 (−31.3%)	−0.719 (+45.1%)
$2_2 - 2_2$	1.917 (−9.6%)	1.795 (−14.5%)	1.627 (+6.3%)	1.4115 (+43.3%)
$4_2 - 4_2$	−1.179 (−5.3%)	−1.136 (+6.9%)	−1.102 (+15.6%)	−0.826 (+0.5%)
$6_2 - 6_2$	−2.168 (∅)	−1.938 (−45.7%)	−0.818 (−2.2%)	−0.751 (+44.4%)
$8_2 - 8_2$	−2.547 (∅)	−2.181 (∅)	−1.484 (−41.3%)	−0.999 (−9.7%)

NOTE: major deviations are for smallest M.E.'s (destructive interference)

A triaxial rotor model:

case study--the Os isotopes

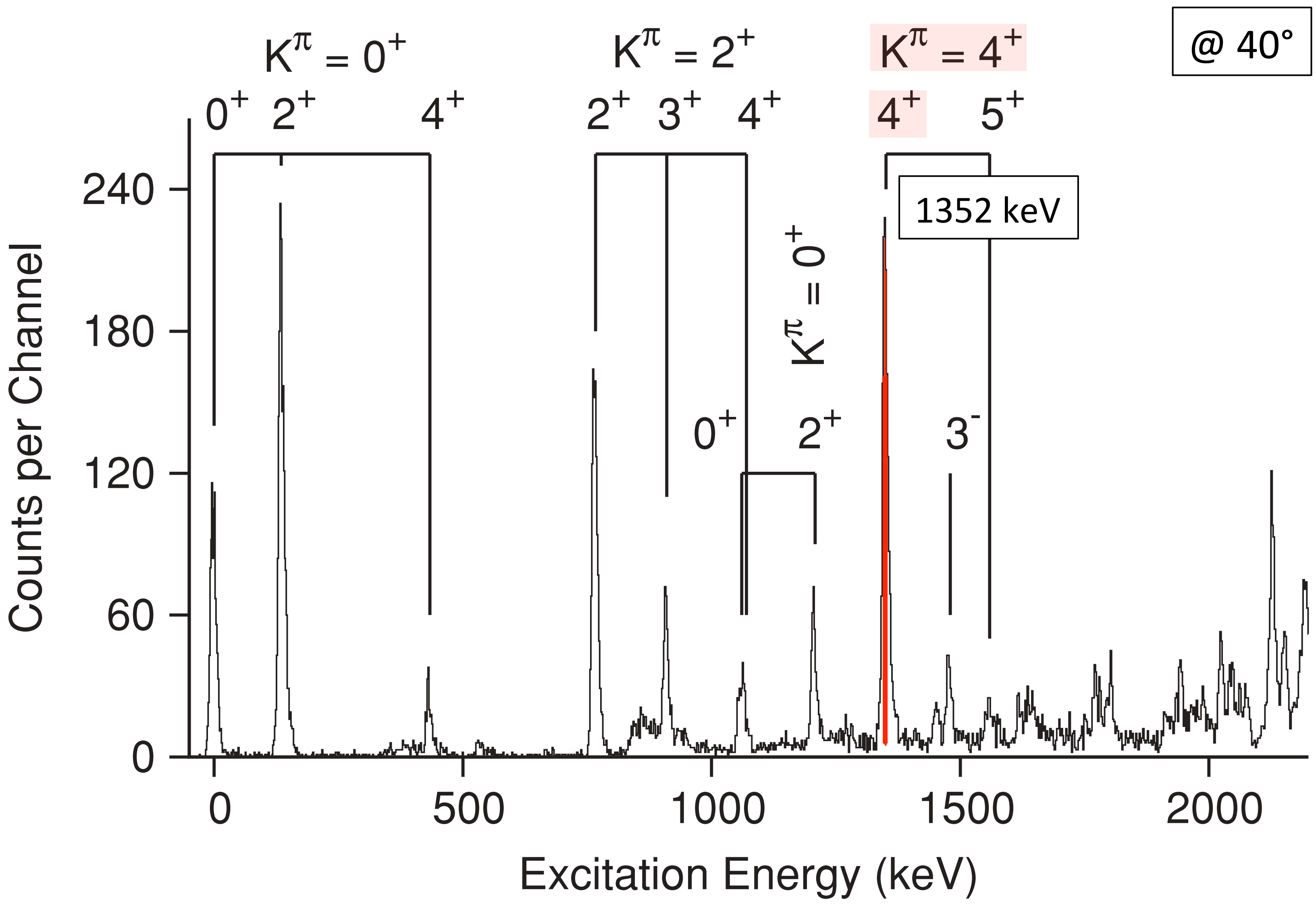
<E2> eb calc. (% dev. from expt.)—full diagonalization Allmond et al.

	^{186}Os	^{188}Os	^{190}Os	^{192}Os
$4_3 - 2_1$	~0.000(−100.4%)	0.0058 (−95.2%)	0.0395 (−24.1%)	0.0916 (−20.4%)
$4_3 - 2_2$	0.7971 (−33.0%)	0.677 (−18.4)	0.554 (−28.1%)	0.4206 (−46.5%)
$4_3 - 3_1$	0.975 (−35.8%)	0.970 (−17.1%)	1.217 (−21.5%)	1.433 (−12.1%)
$4_3 - 4_2$	0.899 (−50.8%)	0.975 (−40.5%)	1.468 (−7.6%)	1.960 (+64.7%)
$4_3 - 4_3$	3.397 (+44.5%)	3.153 (+17.7%)	2.678 (+162.6%)	1.930 (+50.8)

BUT...

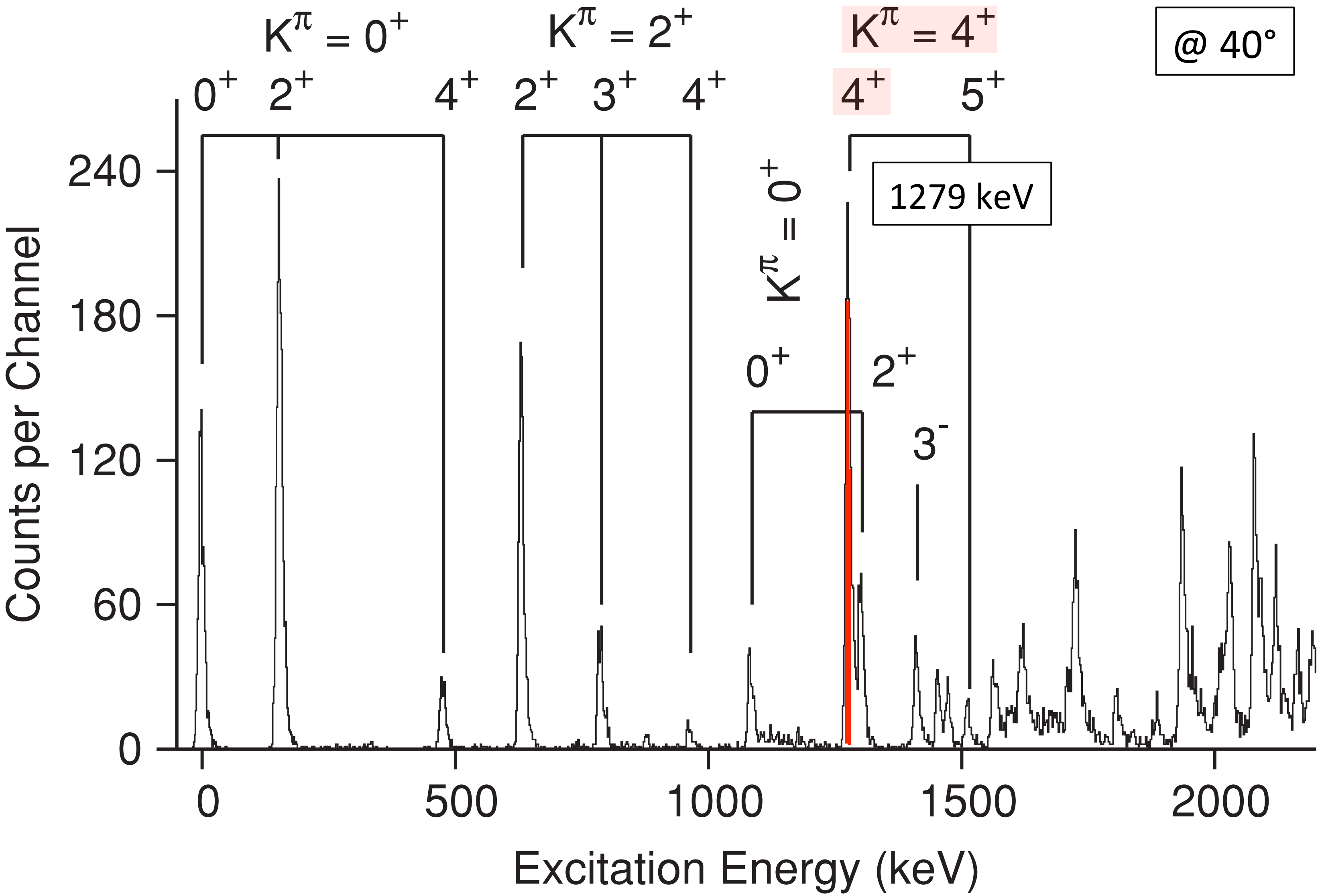
$^{185}Re(^3He,d)^{186}Os$

A. Phillips, P.E. Garrett et al.
PR **C82** 034321 (2010)

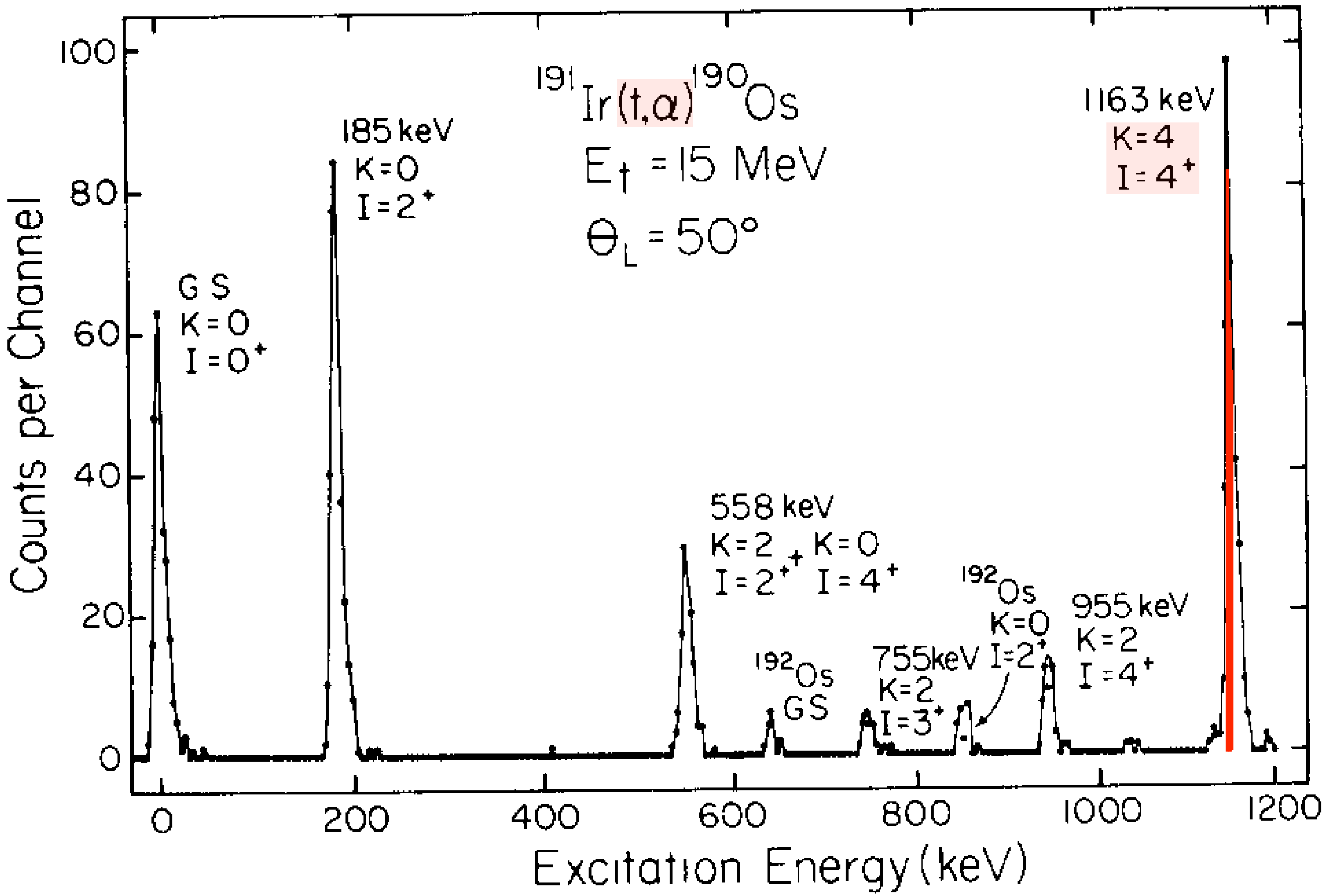


$^{187}Re(^{3}He,d)^{188}Os$

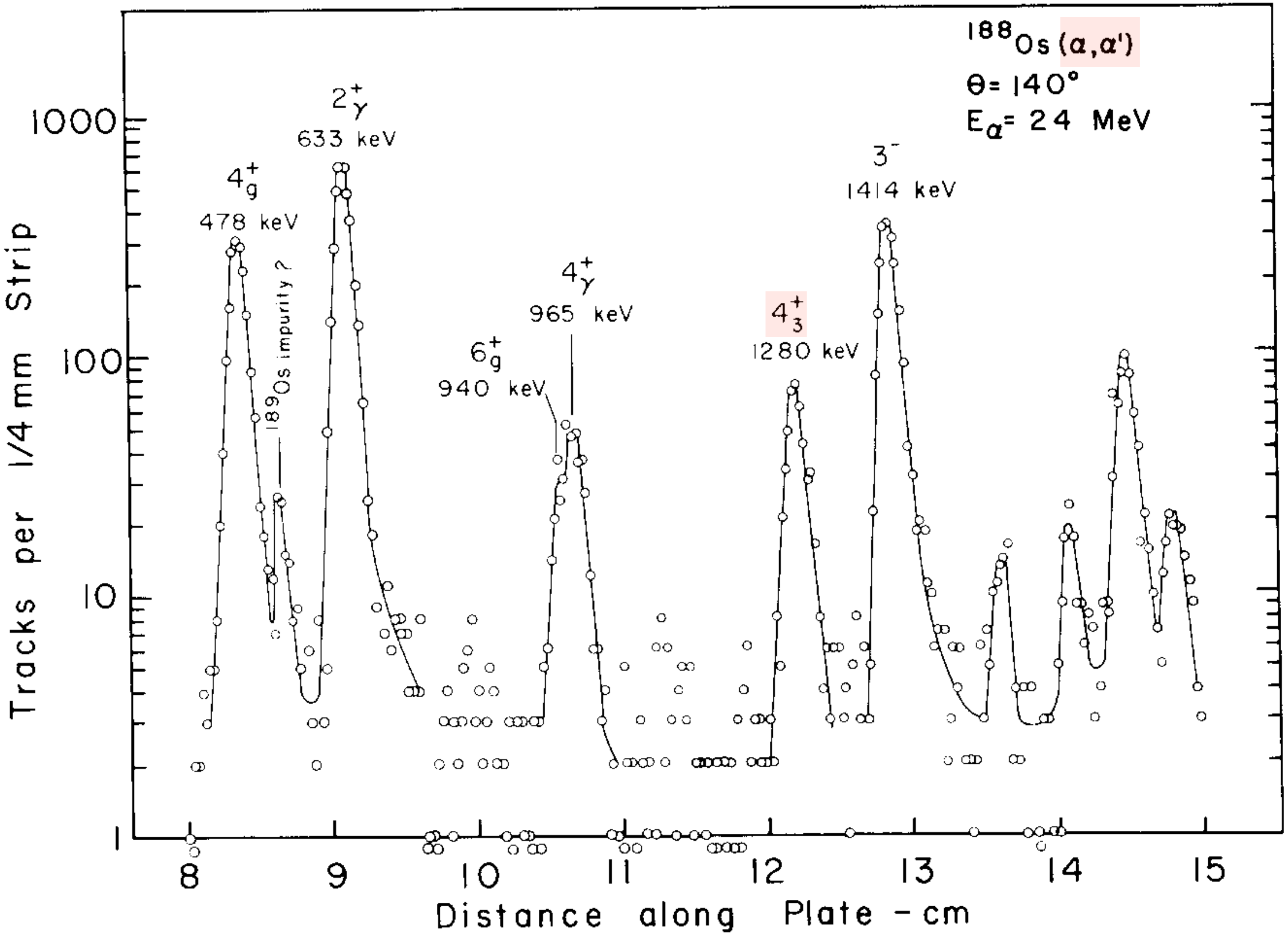
A. Phillips, P.E. Garrett et al.
PR **C82** 034321 (2010)



R.D. Bagnell,… D.G. Burke
PL **B66** 129 (1977)

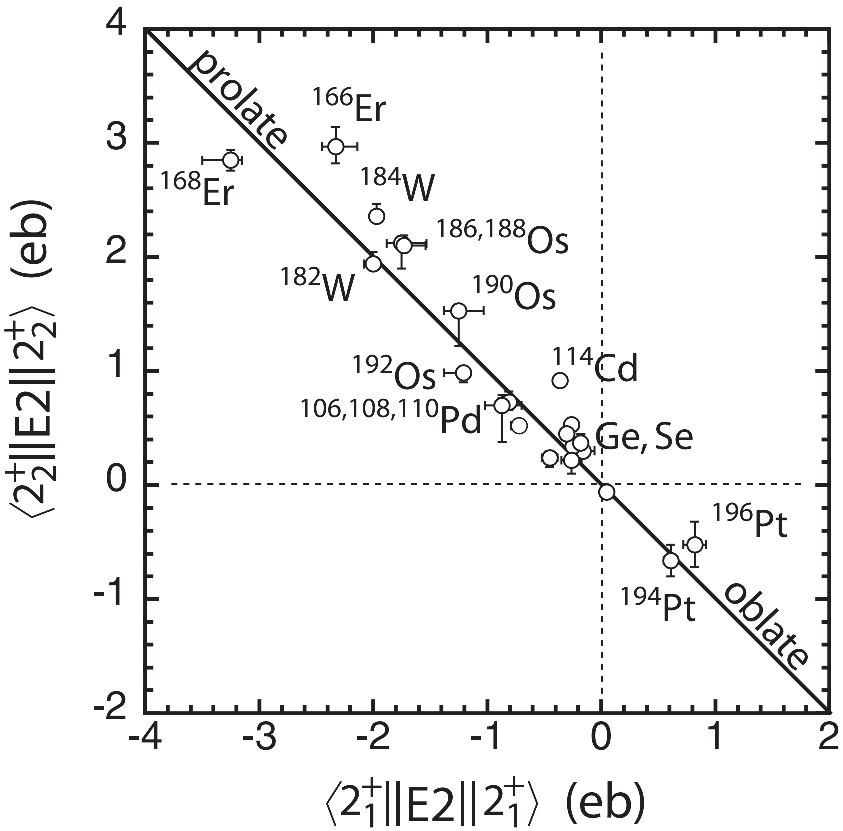


D.G. Burke et al.
PL **B78** 48 (1978)



E2 diagonal sum rules:
spin 2

$$\langle 2_1^+||E2||2_1^+\rangle + \langle 2_2^+||E2||2_2^+\rangle \overset{\star}{\sim} \{\langle 20;20|20\rangle + \langle 22;20|22\rangle\} = 0$$



★ If K is a good quantum number

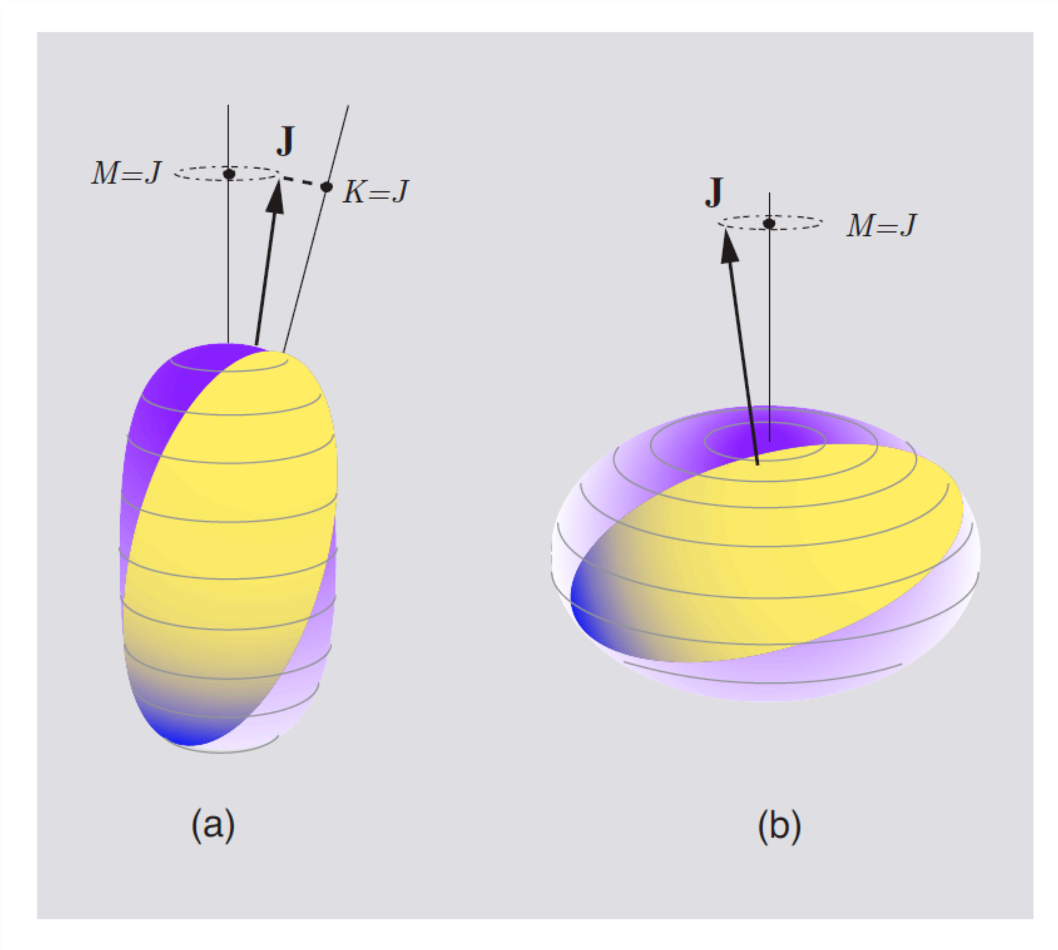
CONCLUSION:
NO MISSING STRENGTH

$$Q(\alpha K I) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} e\bar{Q}_0(\alpha K)$$

Sign change for I = 2:
K = 0 cf. K = 2

J.M. Allmond, PR **C88** 041307R (2013)

Nuclear rotation: (reminder)
laboratory frame vs. body frame

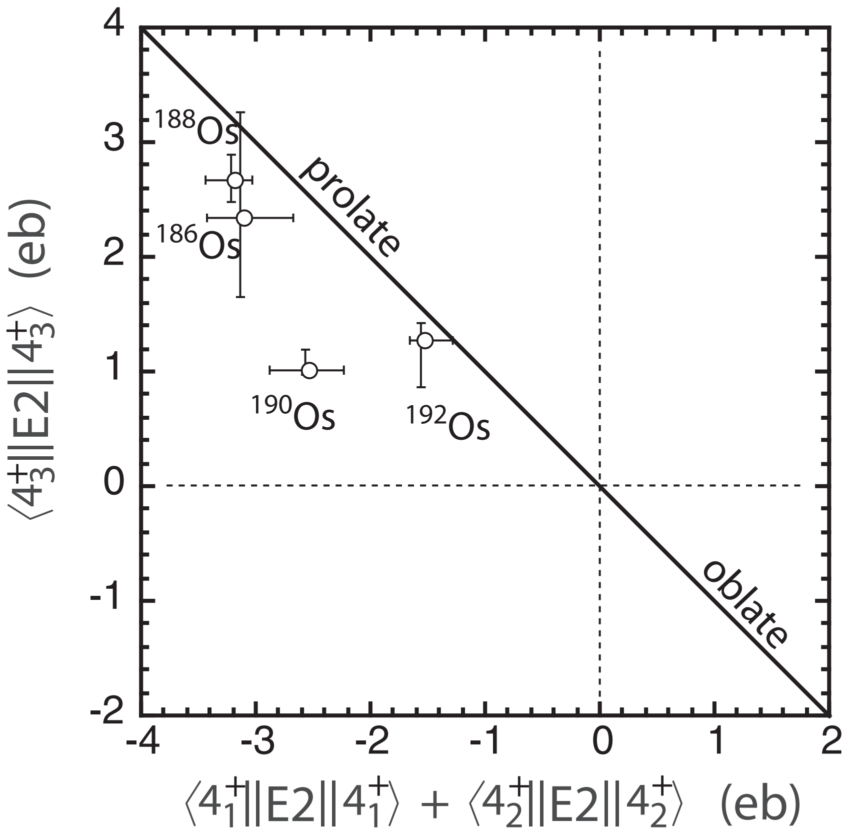


Prolate rotor: (a) K = J (b) K = 0

R&W Fig. 1.45

E2 diagonal sum rules:
spin 4

$$\langle 4_1^+||E2||4_1^+\rangle + \langle 4_2^+||E2||4_2^+\rangle + \langle 4_3^+||E2||4_3^+\rangle \overset{\star}{\sim} \{\langle 40;20|40\rangle + \langle 42;20|42\rangle + \langle 44;20|44\rangle\} = 0$$



★ If K is a good quantum number

CONCLUSION:
MISSING STRENGTH

$$Q(\alpha KI) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} e\bar{Q}_0(\alpha K)$$

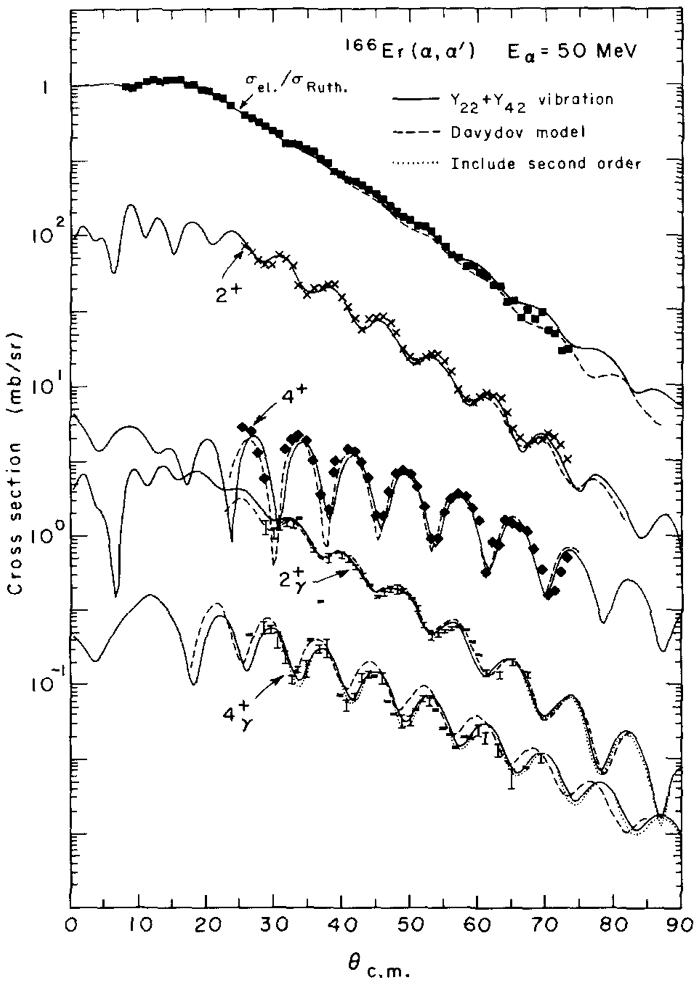
Sign change for I = 4:
K = 0, 2 cf. K = 4

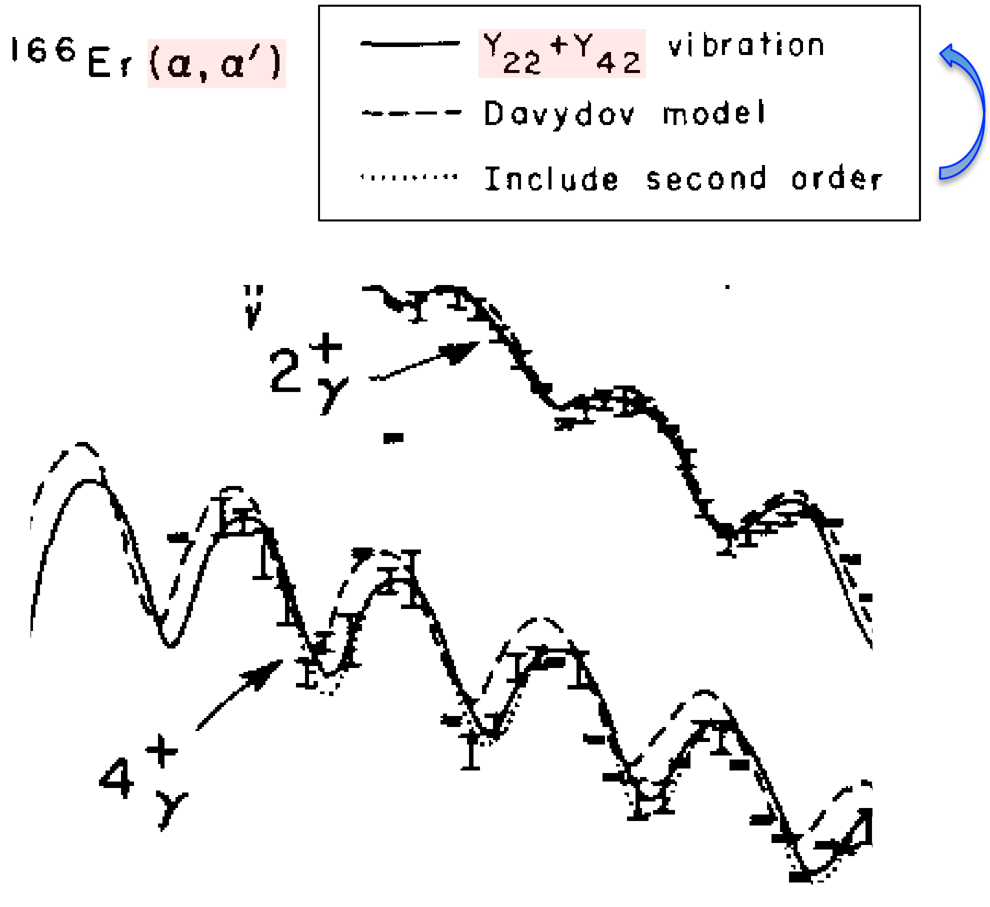
J.M. Allmond, PR **C88** 041307R (2013)

Hexadecapole collectivity:

manifestation in K = 2 (gamma) bands—

a critical message from 50 years ago--forgotten





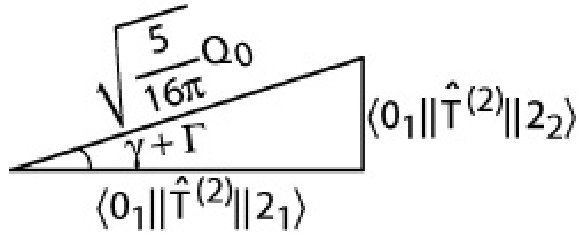
R.S. Mackintosh, PL **B29** 629 (1969)

A triaxial rotor model:

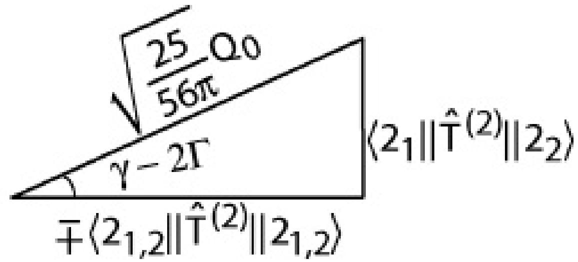
with independent electric quadrupole and inertia tensors

- Kumar-Cline sum rules

$$\boxed{\langle q^2\rangle \equiv \langle 0_1^+\|\hat{Q}\|2_1^+\rangle\langle 2_1^+\|\hat{Q}\|0_1^+\rangle + \langle 0_1^+\|\hat{Q}\|2_2^+\rangle\langle 2_2^+\|\hat{Q}\|0_1^+\rangle}$$



$$\boxed{\langle q^3 \cos 3\delta\rangle \equiv \sum_{r,s=1,2} \langle 0_1^+\|\hat{Q}\|2_r^+\rangle\langle 2_r^+\|\hat{Q}\|2_s^+\rangle\langle 2_s^+\|\hat{Q}\|0_1^+\rangle.}$$



$$\begin{aligned}
\langle q^3 \cos 3\delta\rangle &= Q_o^3 \cos^2(\gamma+\Gamma)\cos(\gamma-2\Gamma)\\
&\quad + 2Q_o^3 \cos(\gamma+\Gamma)\sin(\gamma-2\Gamma)\sin(\gamma+\Gamma)\\
&\quad - Q_o^3 \sin^2(\gamma+\Gamma)\cos(\gamma-2\Gamma)\\
&= Q_o^3[\cos(\gamma-2\Gamma)\cos(2\gamma+2\Gamma)\\
&\quad + \sin(\gamma-2\Gamma)\sin(2\gamma+2\Gamma)]\\
&= Q_o^3 \cos[(\gamma-2\Gamma)+(2\gamma+2\Gamma)]\\
&= Q_o^3 \cos 3\gamma,
\end{aligned}$$

K. Kumar, PRL **28** 249 (1972)

D. Cline, Ann. Rev. Nucl. Part. Sci. **36** 683 (1986)

J.L. Wood et al.
PR **C70** 024308 (2004)

A triaxial rotor model:

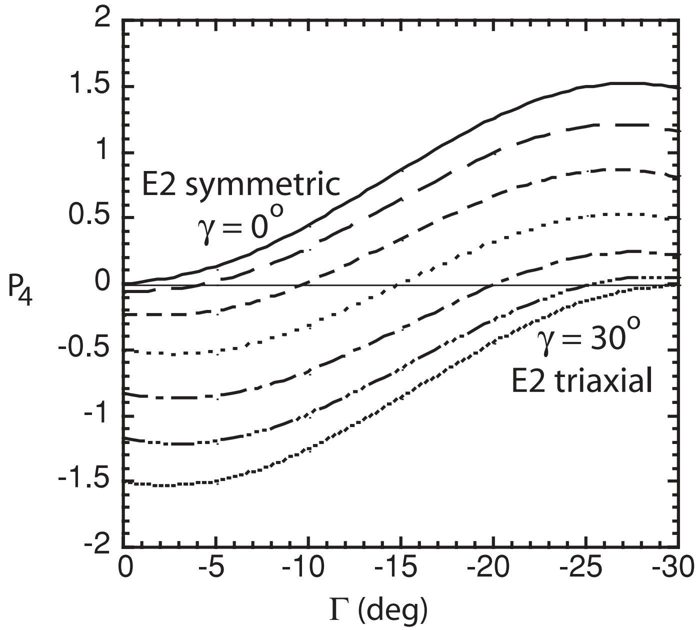
with independent electric quadrupole and inertia tensors

- P_3 and P_4 interference terms:

$$\boxed{P_3 = \langle 0_1 || \hat{T}(E2) || 2_1 \rangle \langle 2_1 || \hat{T}(E2) || 2_2 \rangle \langle 2_2 || \hat{T}(E2) || 0_1 \rangle,}$$

$$\boxed{P_4 = \langle 2_1 || \hat{T}(E2) || 2_1 \rangle P_3,}$$

$$\boxed{P_4 = \frac{125}{7168\pi^2} Q_0^4 \left[\cos(4\gamma - 2\Gamma) - \cos 6\Gamma\right],}$$



Lines: $\gamma = 0°, 5°, 10°, 15°, 20°, 25°, 30°$

Plot: without scale factor

All previous models: cannot give $P_4 > 0$.

J.M. Allmond et al.
PR **C80** 021303R (2009)

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

- P_4 term in ^{194}Pt

J.M. Allmond et al.
PR **C80 021303R** (2009)

M.E.	Exp. (e b)	Theory (e b)	% dev.
$\langle 0_1 \vert\vert \hat{T}(E2) \vert\vert 2_1 \rangle$	$(-)\,1.281^{9}$	$(-)\,1.307^{31}$	-2.0%
$\langle 0_1 \vert\vert \hat{T}(E2) \vert\vert 2_2 \rangle$	$(+)\,0.091^{2}$	$(+)\,0.0928^{48}$	2.0%
$\langle 2_1 \vert\vert \hat{T}(E2) \vert\vert 2_2 \rangle$	$(-)\,1.53^{5}$	$(-)\,1.449^{34}$	5.1%
$\langle 2_1 \vert\vert \hat{T}(E2) \vert\vert 2_1 \rangle$	$+0.61^{6}$	$+0.595^{16}$	-3.1%
$\langle 2_2 \vert\vert \hat{T}(E2) \vert\vert 2_2 \rangle$	-0.66^{14}	-0.595^{16}	9.6%
	Exp. (e b)^4	Theory (e b)^4	% dev.
P_4[a]	$+0.109^{11}$	$+0.105^{7}$	-4.3%

[a]The sign of the P_4 term is independent of all phase-factor conventions for the E2 matrix elements (unlike the P_3 term).

Q_0 = - 4.155 eb, γ = 19.85°, Γ = -23.92°

Data from:
C.Y. Wu, D. Cline, T. Czosnyka, et al.
NP **A607** 178 (1996);
and see refs. In Allmond et al.

$P_4 < 0$ for:
- Davydov model
- anharmonic vibrator model
- pairing-plus-quadrupole model

A triaxial rotor model:

with independent electric quadrupole and inertia tensors

- Destructive interference of E2 matrix elements in a triaxial rotor model: ^{196}Pt

M.E.	Exp. (eb) [4]	Theory (eb)	diff. (eb)	% diff.
$\langle 0_1 \vert\vert \hat{T}(E2) \vert\vert 2_1 \rangle$ [a]	(−)1.172(3)	(−)1.184(28)	−0.012	−1.0%
$\langle 0_1 \vert\vert \hat{T}(E2) \vert\vert 2_2 \rangle$	0.0	0.0	0.0	0.0%
$\langle 2_1 \vert\vert \hat{T}(E2) \vert\vert 2_2 \rangle$ [a]	(−)1.36(5)	(−)1.243(51)	0.117	8.6%
$\langle 2_1 \vert\vert \hat{T}(E2) \vert\vert 2_1 \rangle$	+0.83(9)	+0.676(79)	−0.154	−18.6%
$\langle 2_2 \vert\vert \hat{T}(E2) \vert\vert 2_2 \rangle$	−0.51(21)	−0.676(79)	−0.166	−32.5%

[a] Because the signs of individual off-diagonal or transitional $E2$ matrix elements are not directly observable (unlike the diagonal $E2$ matrix elements), a negative sign, (−), is adopted here for $\langle 0_1||\hat{T}(E2)||2_1\rangle$ and $\langle 2_1||\hat{T}(E2)||2_2\rangle$ to comply with the $-\beta_2 \propto -Q_0$, $\gamma = 0^\circ - 30^\circ$ convention [3].

Q_0 = - 3.754 eb, γ = 20.5°, Γ = -20.5°

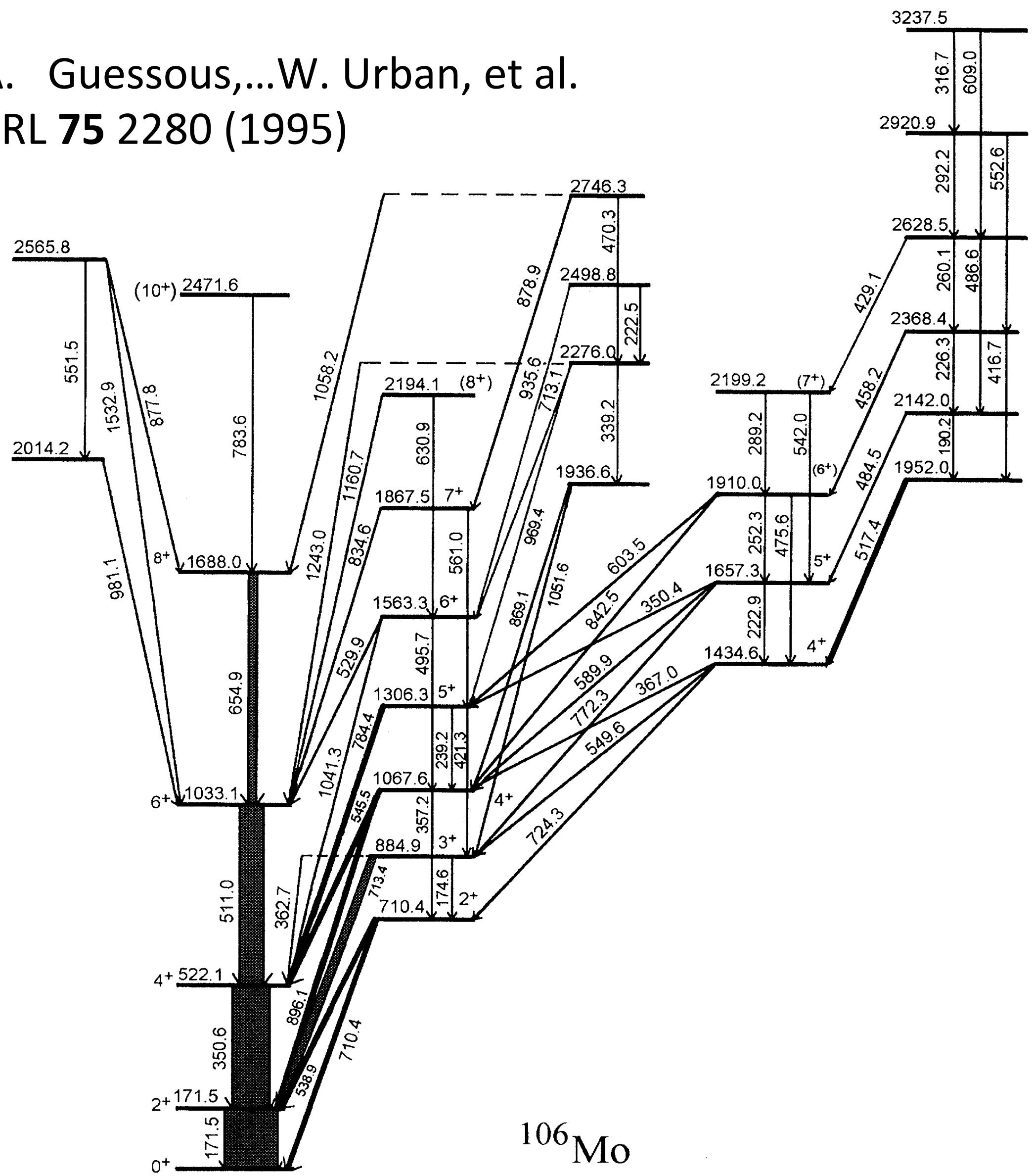
Exp.: NDS; C.S. Lim et al., NP **A548** 308 (1992)

NOTE: "O(6)" limit of the IBM gives:
$<0_1||T(E2)||2_2> = 0$ and $<2_1||T(E2)||2_1> = 0$

J.M. Allmond et al.,
PR **C81** 051305R (2010)

Triaxiality?

A. Guessous,…W. Urban, et al.
PRL **75** 2280 (1995)



CONCLUSIONS

- Need more data, e.g., A ~ 106 region, to answer questions about triaxiality.
- Coulex studies MUST be complemented by transfer reaction and inelastic scattering spectroscopy.
- As a Community, we are running a grave risk of "forgetting" critical (historical) data.

CONTENTIONS

- We need a much better perspective on nuclear "rotation" and what we mean by "moment of inertia".
- There is a serious lack of "standard" spectroscopy*:

"fashion" is far too prevalent (and cannot answer fundamental open questions).

*And lack of facilities to conduct these critical experiments.