Correlated Error Analysis in GOSIA2

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GOSIA Workshop 2018 – 9th April 2018

Radioactive Ion Beam Coulex

- Advent of **R**adioactive **I**on **B**eams (**RIB**s) leads to exciting new physics.
- Coulex has large cross-sections and sensitive to key nuclear-structure info!

!!! BUT !!!

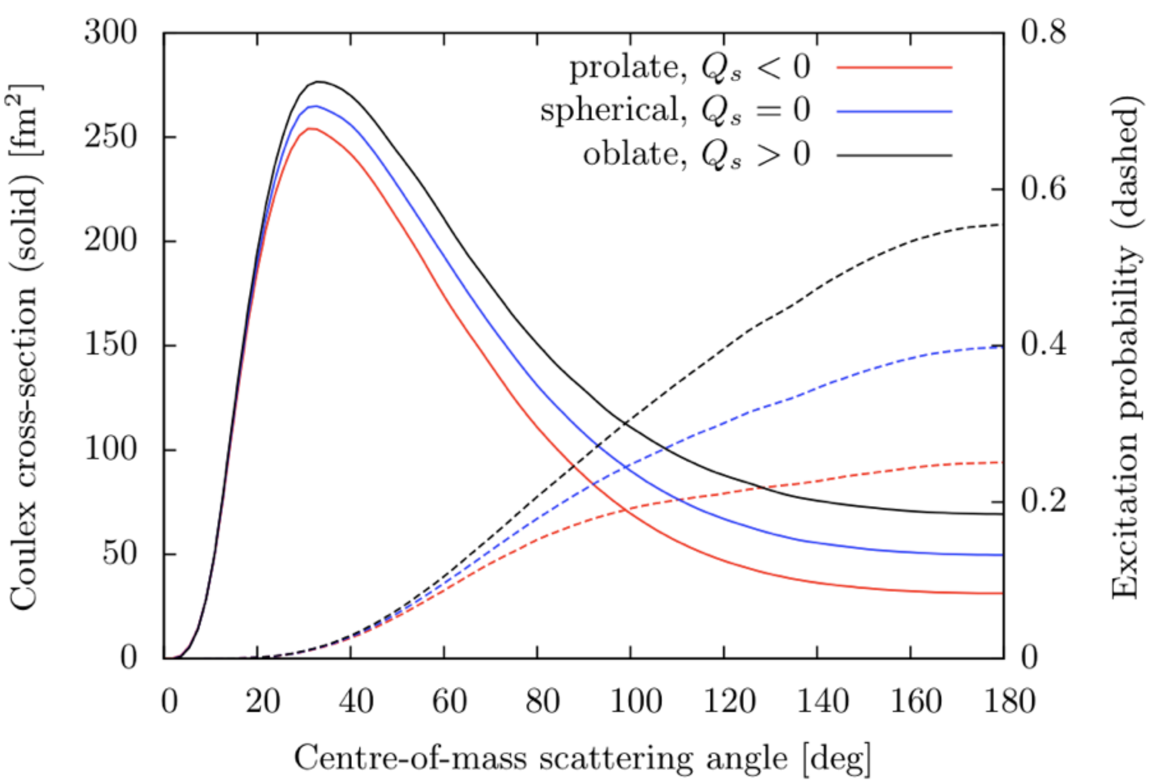
✓ No spectroscopic data

✓ Low statistics

✓ Few data points

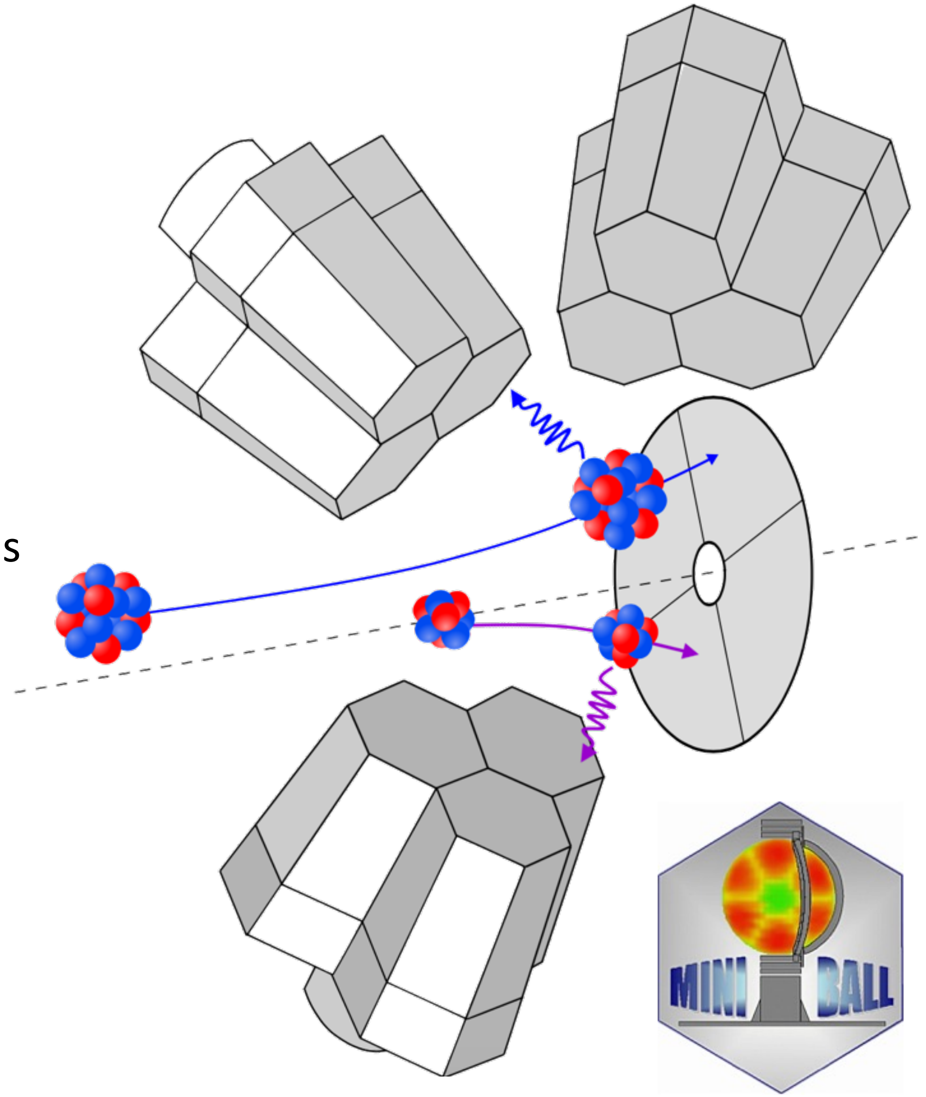
= **Under-determination of Gosia fit!**

Magda described this!



Miniball Coulex set-up

- Particle detector at forward lab. angles, focused on cross-section.
  - ➢ Inverse kinematics!
- Recoil information gives more backwards c.o.m angles.
- Rutherford normalisation becomes *extremely* sensitive to angle.
- Downscaling, p-γ efficiency, etc. causes further problems.
- Solution, normalise to target!
  - ➢ All conditions identical
  - ➢ Only Doppler correction changed



N. Warr et al., EPJ **49** (2013)

GOSIA2 – Target normalisation

- Stable target species
  - Known matrix elements
  - Therefore, known cross-section

$$N_t = L \cdot \frac{\rho d N_A}{A_t} \cdot b_t \epsilon_\gamma(E_t) \epsilon_{\text{part}} \sigma_t$$

- Usually choose low XS target (***t***)
  - Clean γ-ray spectrum
  - Low detector rates

$$N_p = L \cdot \frac{\rho d N_A}{A_t} \cdot b_p \epsilon_\gamma(E_p) \epsilon_{\text{part}} \sigma_p$$

- Can be used to get absolute XS of projectile (***p***) in relative measurement

- Removes systematic effects:
  - Target thickness
  - Particle-γ efficiency
  - Beam intensity

$$\frac{N_p}{N_t} = \frac{b_p \epsilon_\gamma(E_p) \sigma_p}{b_t \epsilon_\gamma(E_t) \sigma_t}$$

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Knowns
Unknowns

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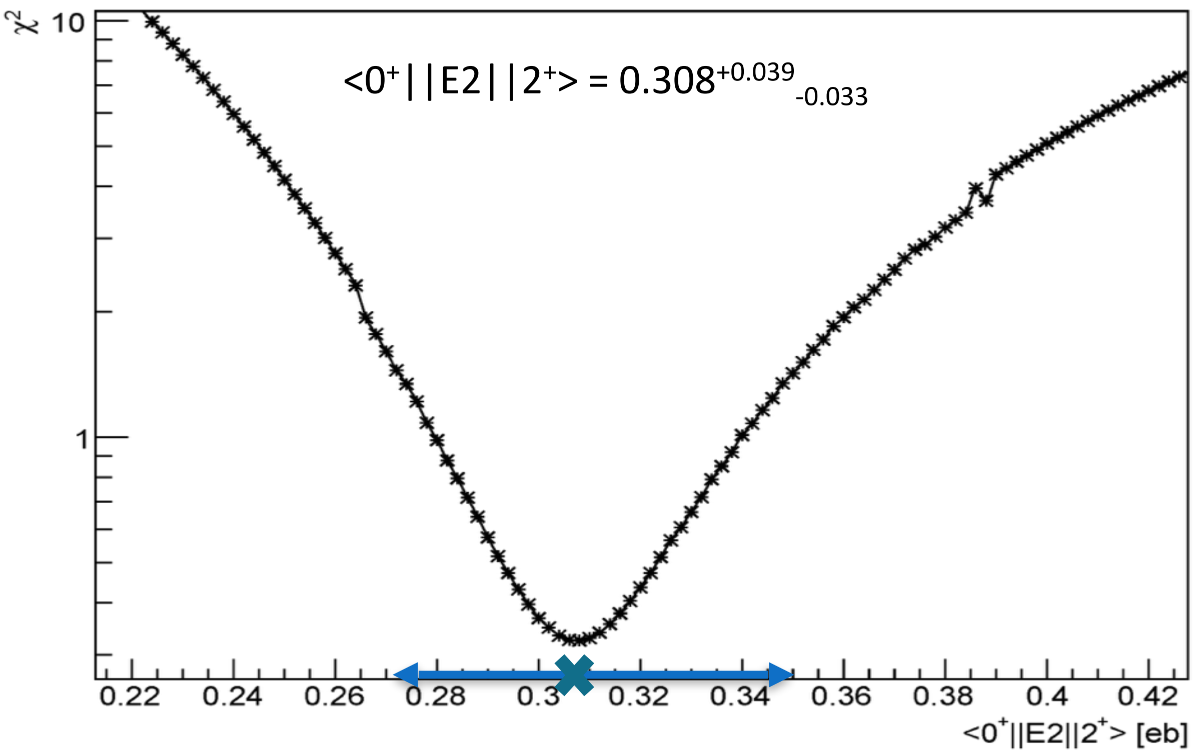
"Knowns" have errors too!

Knowns
Unknowns

Cross-section = B(E2)?

- GOSIA2: Fit target excitation as normalisation:
  - Standard OP,MINI gives best fit, but OP,ERRO neglects target system
  - χ^2 scan of transitional matrix element in projectile
  - Sum of total χ^2 from *target* and from *projectile* systems
  - 1σ error from χ^2+1 method (can be discussed!)



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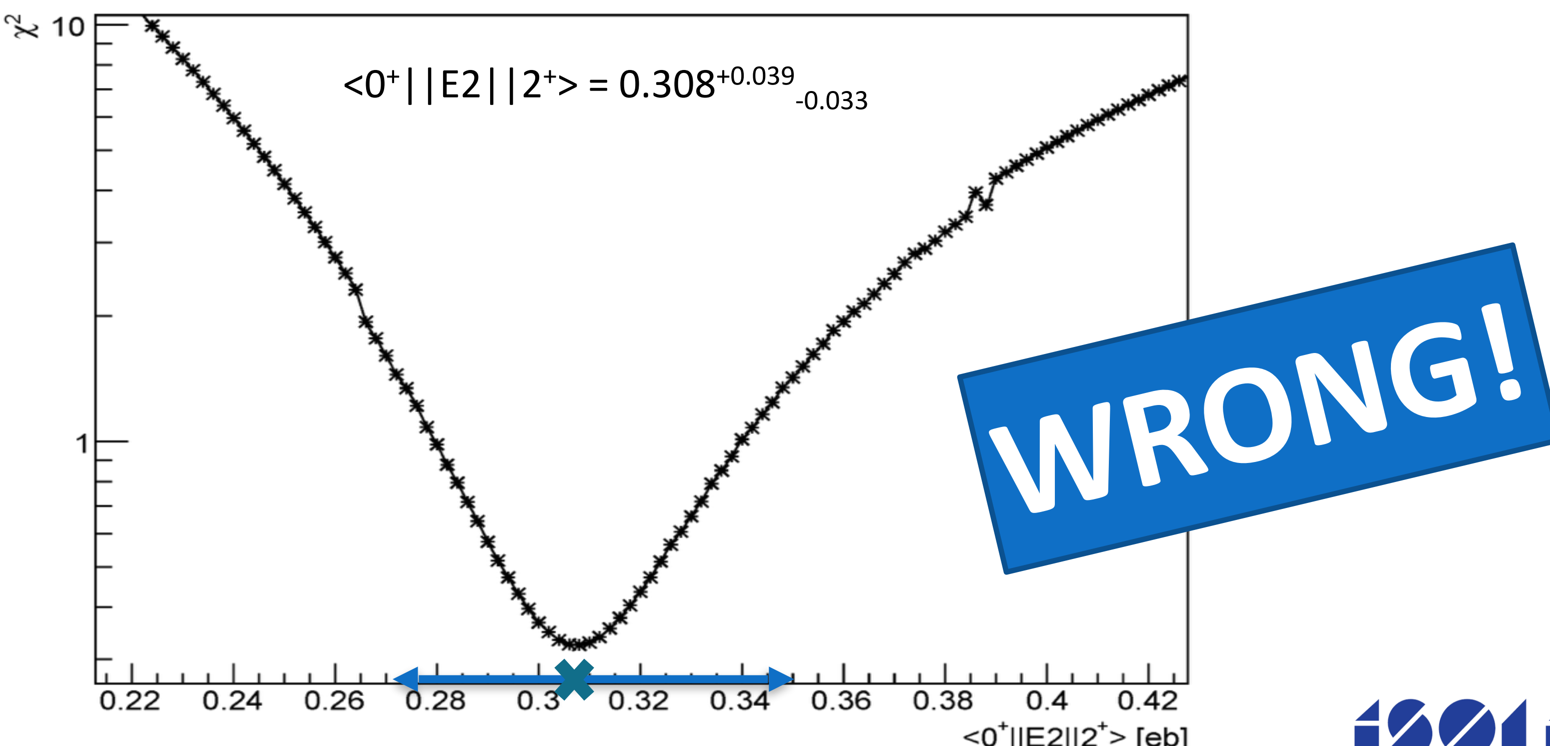
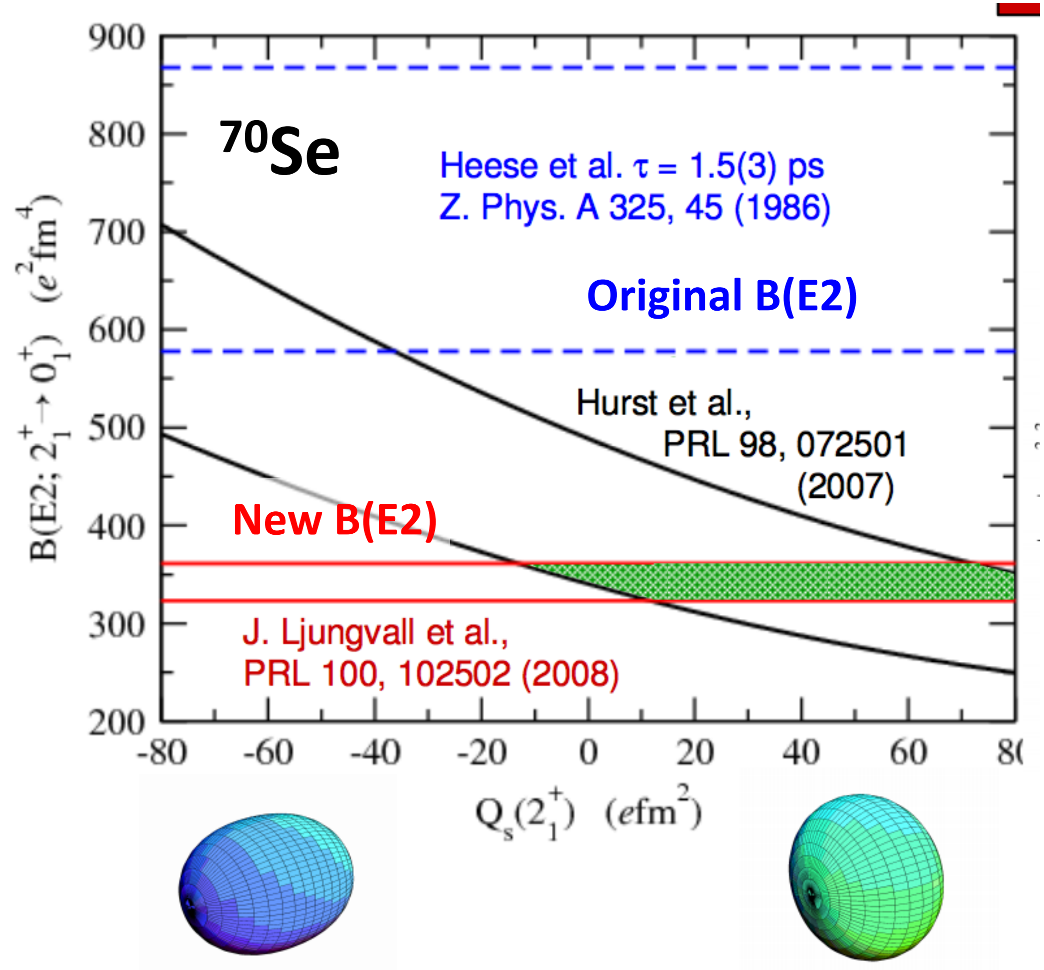


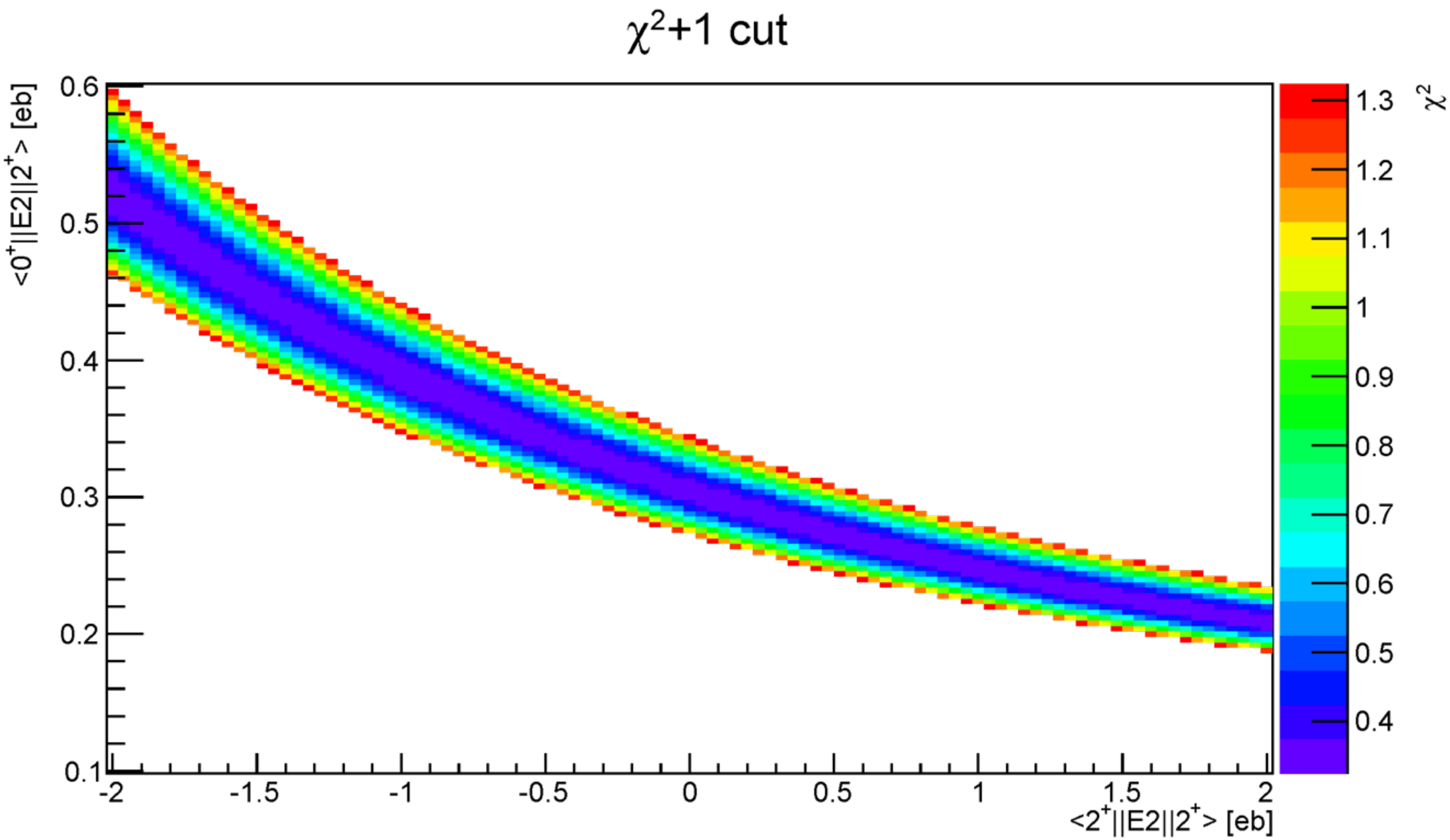
B(E2) vs. $Q_s(2^+)$

- Famous REX-ISOLDE case of ^{70}Se → Measured $Q_s(2^+_1)$
- Strong correlation with 2[nd] order processes (i.e. reorientation)
- Coulex is sensitive to $Q_s(2^+_1)$
  - Good – yes.
  - Bad – yes.
- One Coulex cross-section measurement…
  - Normalise to target
  - Fit still unconstrained!



ISOLDE

χ^2 surface analysis – ^{62}Fe



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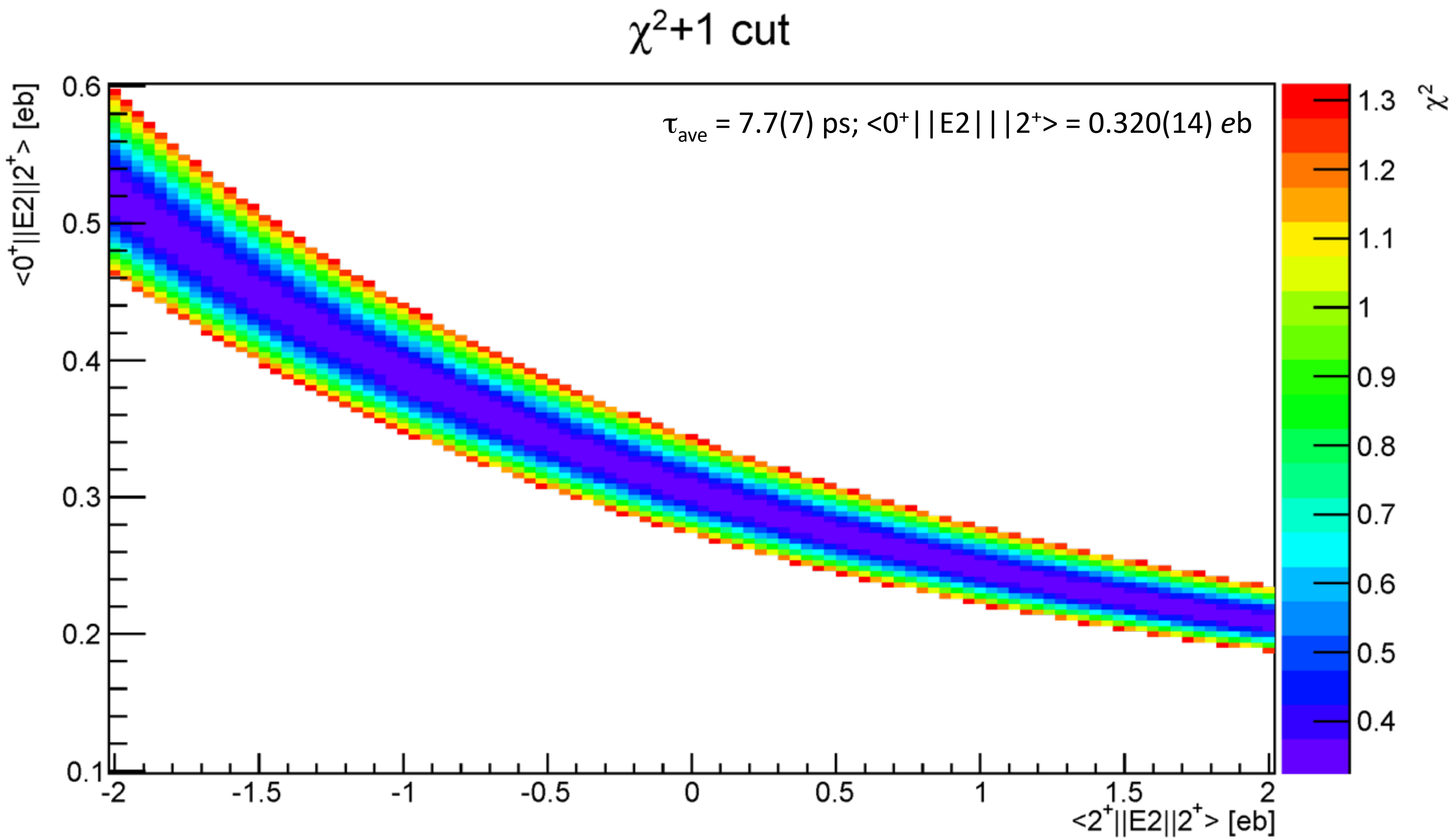

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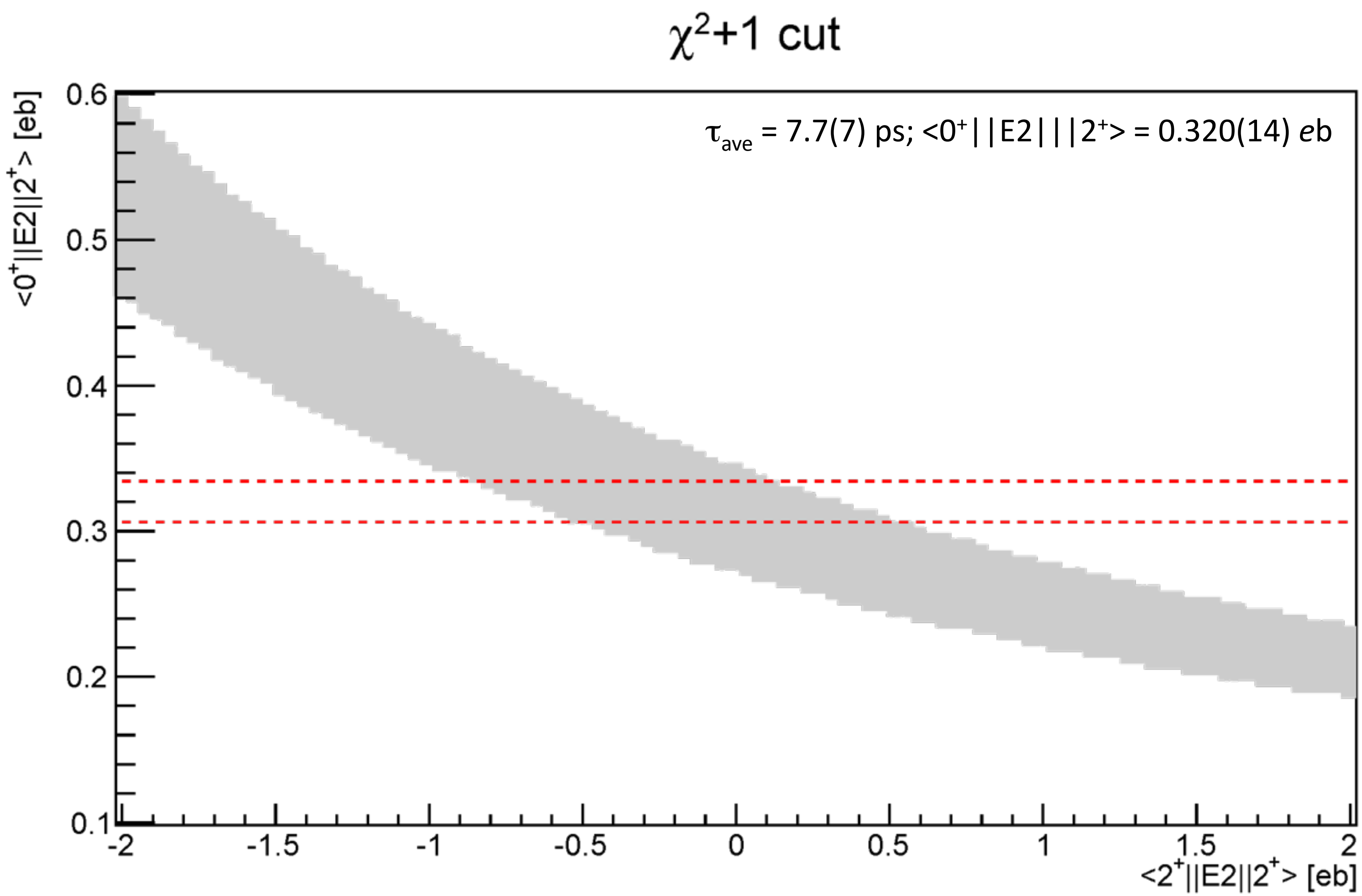

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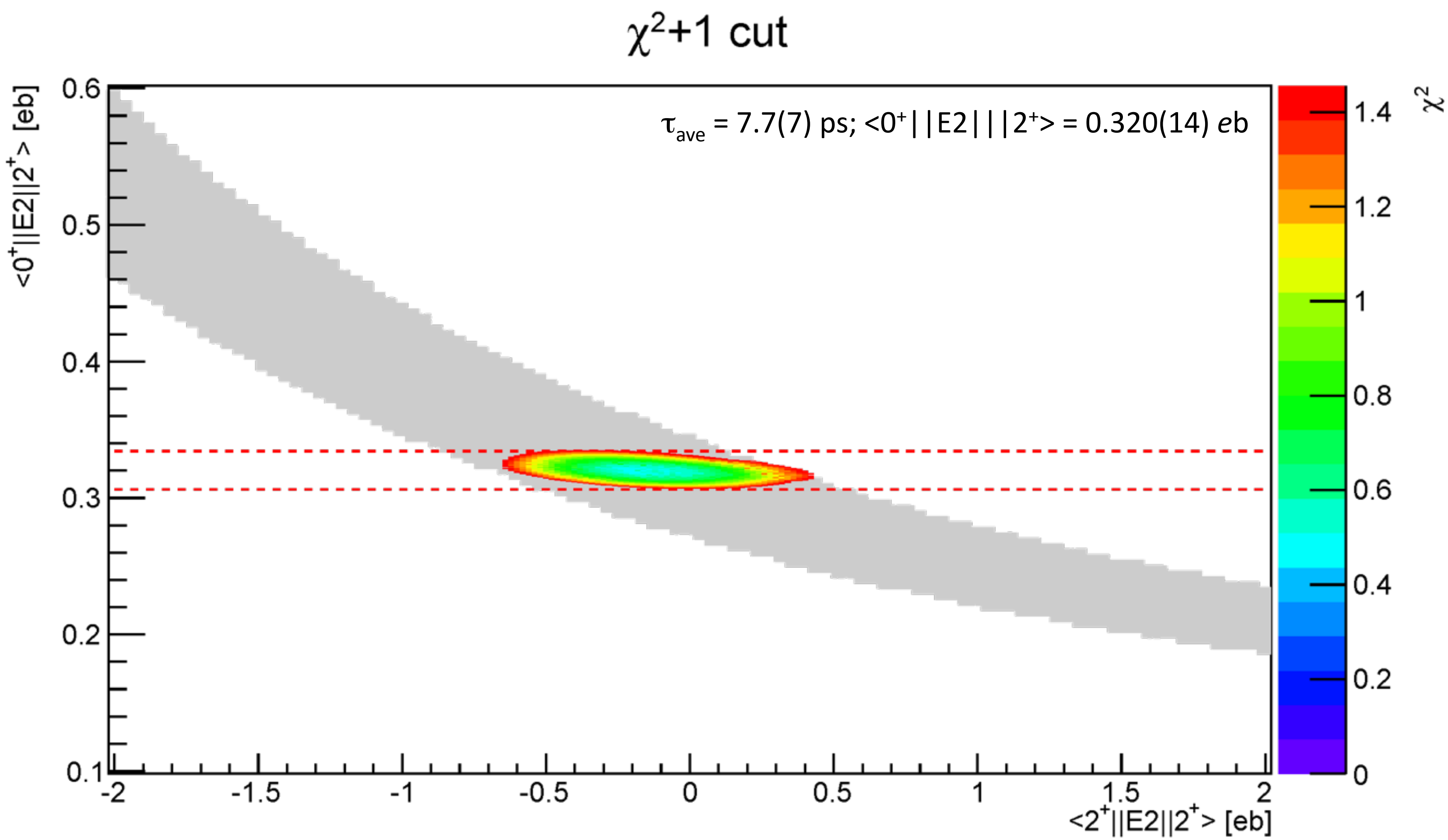

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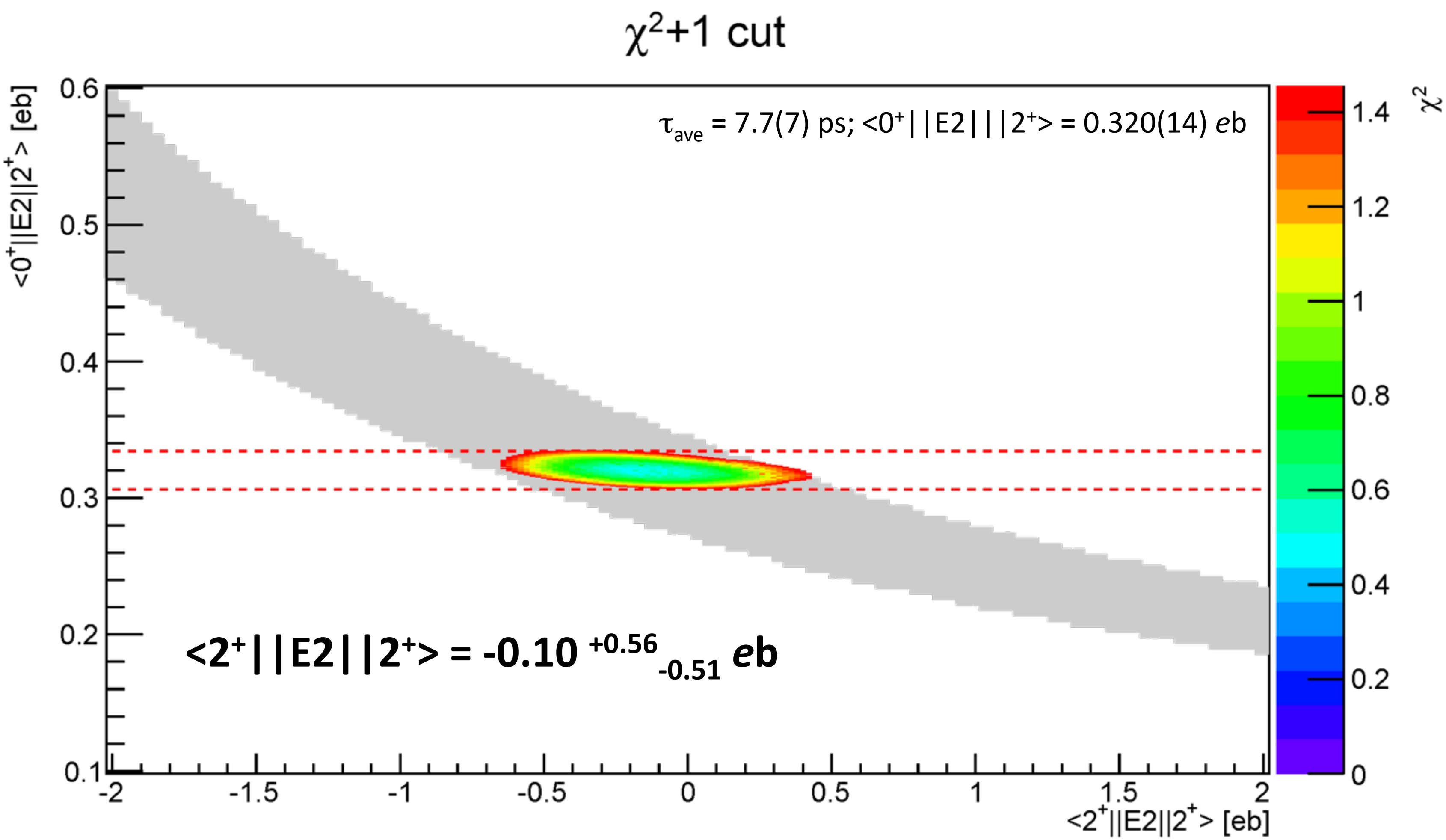
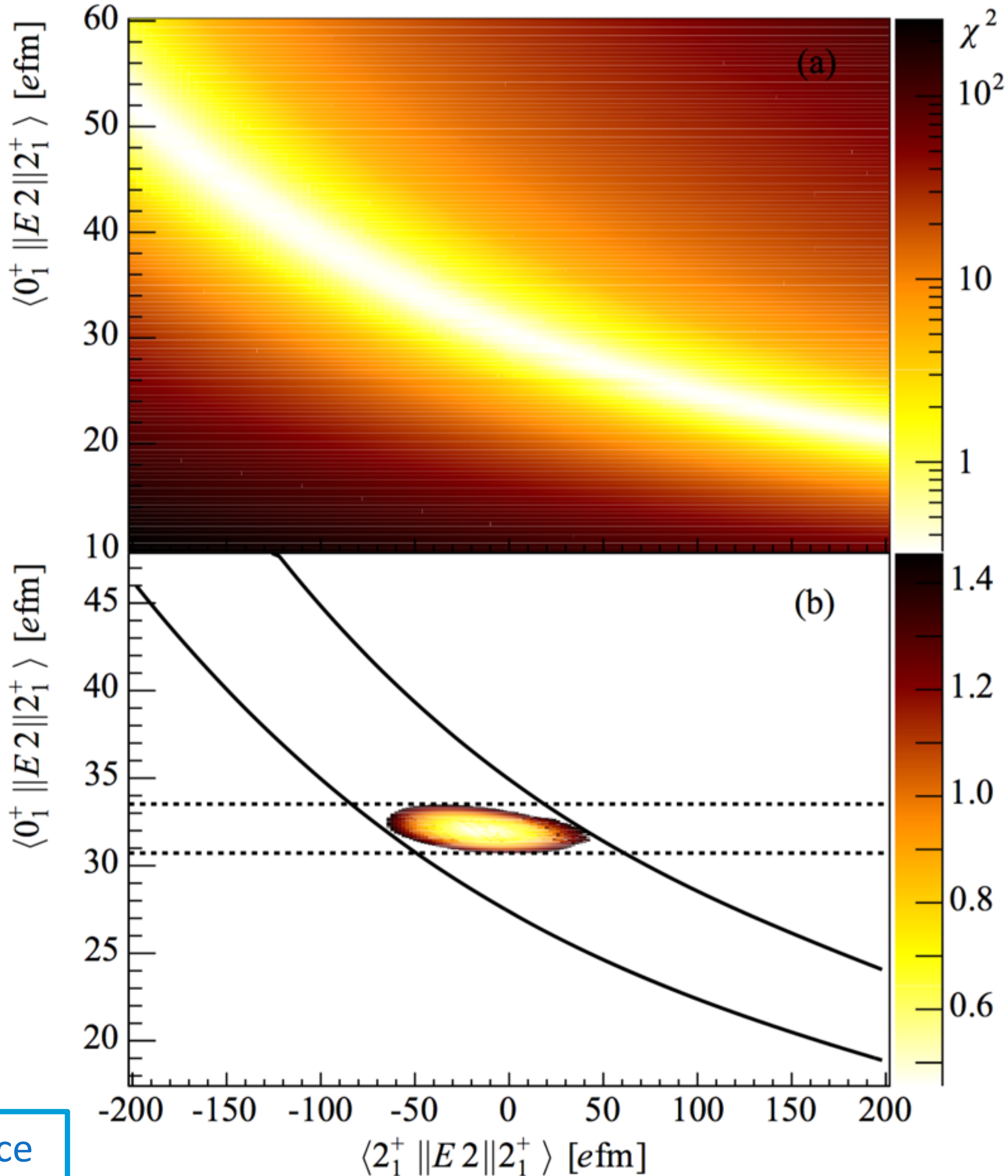


χ^2 surface analysis – ^{62}Fe

- Using $\chi^2 + 1$ method to cut the 1σ surface…
- Coulex alone gives no contraint to fit...
  - One γ-ray yield
  - Two parameters
- Add lifetime information to contstrain fit and extract Q_s
- More Coulex data is better!
  - Angular ranges
  - Different targets
  - Beam energies

https://github.com/lpgaff/chisqsurface



chisqsurface

Each file required for projectile and for target.

- AX = e.g. ^{62}Fe for proj.
  ^{109}Ag for targ.
- yyy = comment/identifier
- Extensions are hardcoded

Filename	Function	Runtime
AX_yyy.inp	OP,MINI	Main input file
AX_yyy.MAP.inp	OP,MAP	Run once
AX_yyy.INTI.inp	OP,INTI	Run each step (proj.) Run once (targ.)
AX_yyy.bst	*Fitted* matrix elements	Called & updated by GOSIA2
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AX_yyy.yld	OP,CORR	γ-ray yields!

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Target OP,MINI → FULL!!

```
OP,REST
0,0
OP,MINI
 2100,10,.0001,.0001,1.1,1,10,1,1,0.0001
OP,MINI
 2100,10,.0001,.0001,1.1,1,10,1,1,0.0001
OP,EXIT
```

Projectile OP,MINI → χ^2 calculator

```
OP,REST
0,0
OP,MINI
 2100,2,99999999.,.0001,1.1,1,10,1,1,0.0001
OP,EXIT
```

chisqsurface

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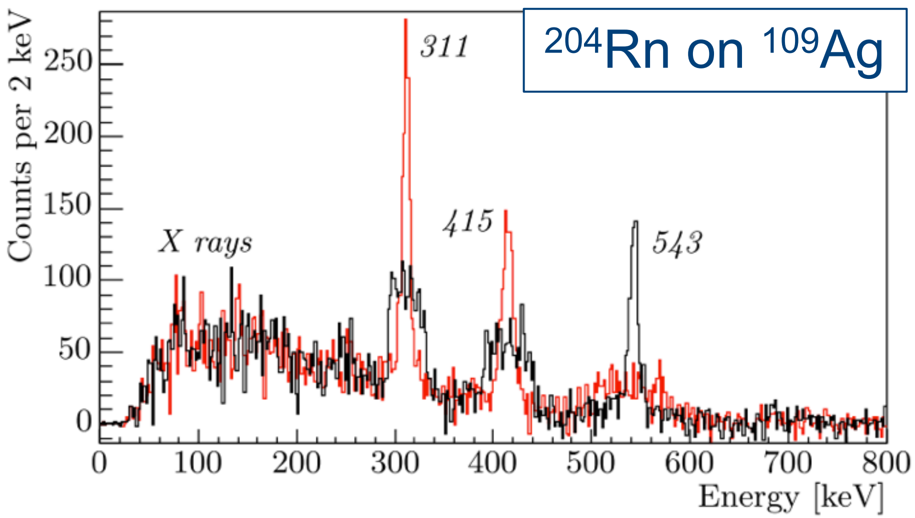
```
Usage:
chisqsurface <in_proj> <in_targ> <Ndata_proj=3> <Ndata_targ=3> <low_TME=0.1>
   <upp_TME=2.5> <Nsteps_TME=51> <low_DME=0.0> <upp_DME=0.0> <Nsteps_DME=1>

 where <Ndata_proj=3> and <Ndata_targ=5> are the number of data
for the projectile and target, respectively. This includes the
sum of all g-ray yields, matrix elements, lifetimes, etc.
```

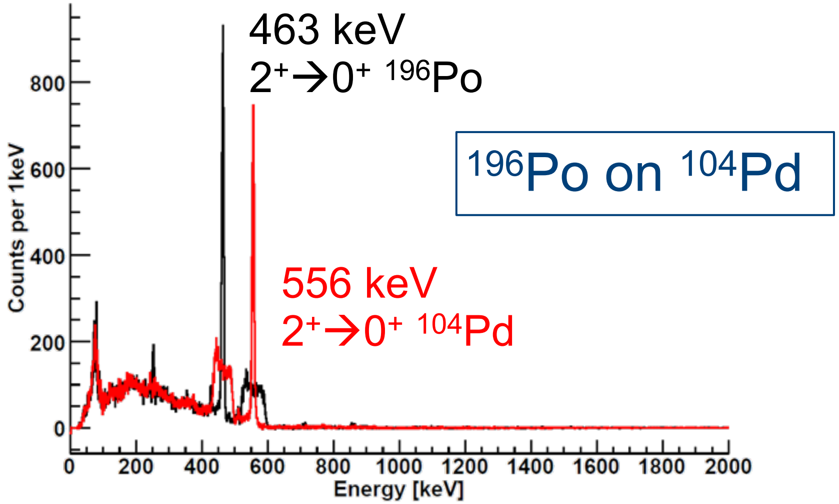
Real-life examples

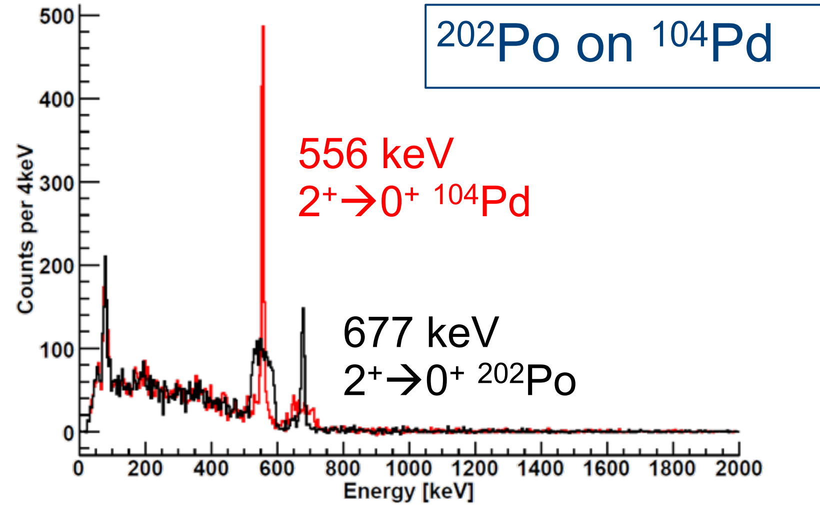
- Target excitation
  - ➢ Known matrix elements!
- Projectile excitation
  - ➢ Single-step dominated
  - ➢ What about two-steps?



Doppler corrected for projectile/target

Real-life examples





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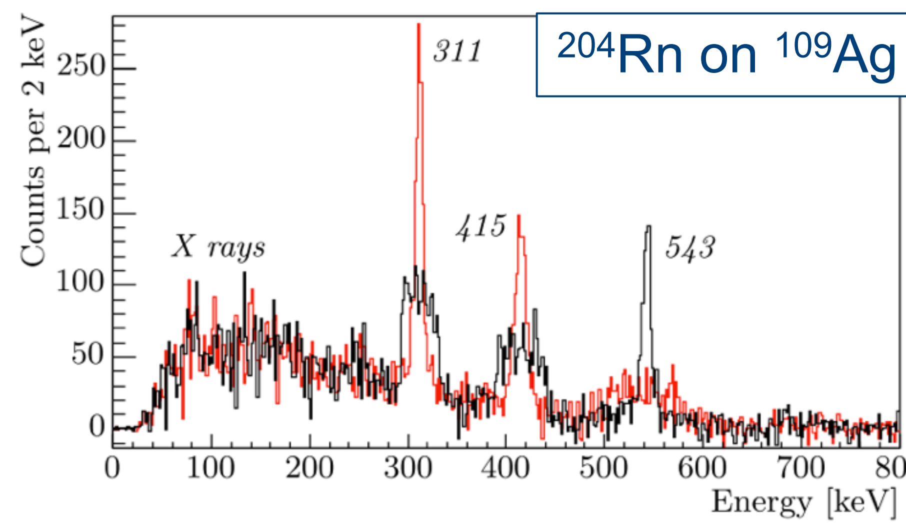
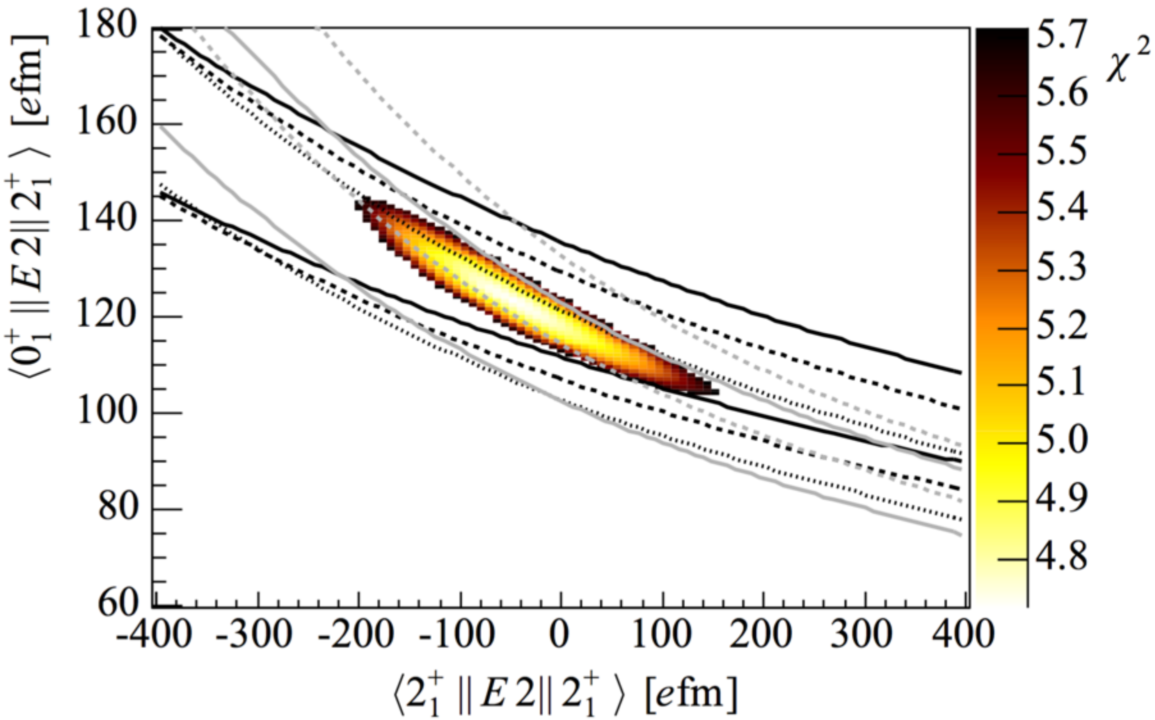
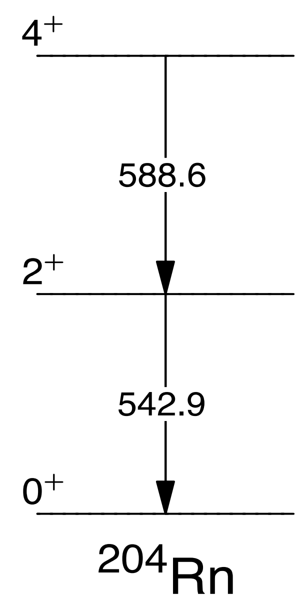


Doppler corrected for projectile/target

Angular ranges – ^{204}Rn

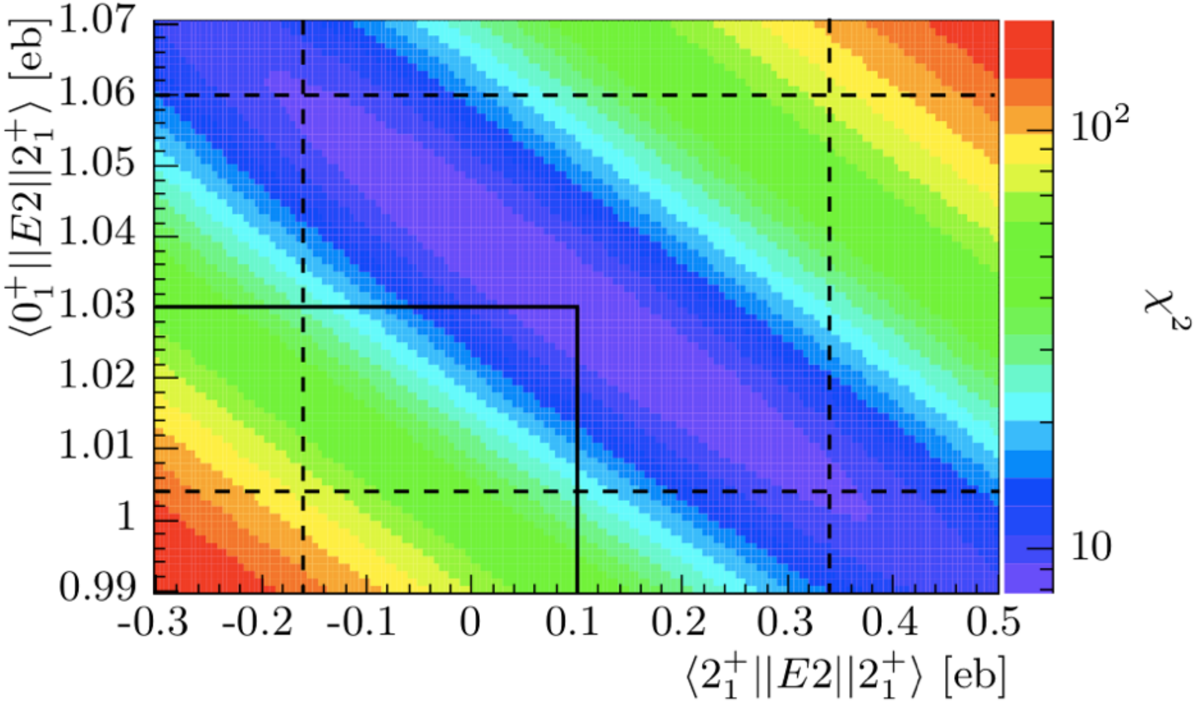
- Five different angular ranges are selected
  - Based on segmentation of Miniball CD detector
- Increasing c.o.m angle leads to increasing $Q_s(2^+_1)$ sensitivity
  - Gradient of χ^2 surface cut gets steeper



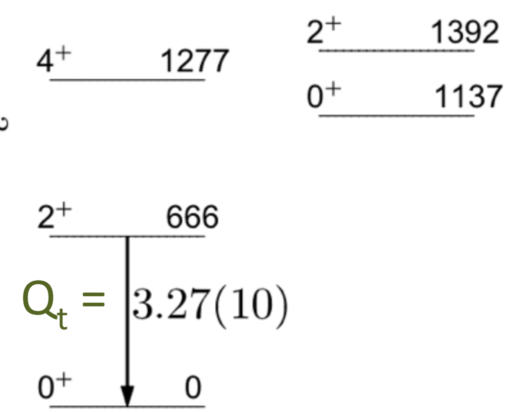


^{200,202}Po – Easy!

- If only 2^+_1 state populated
  - ➢ Extract $\langle 0^+_1||E2||2^+_1\rangle$ and $\langle 2^+_1||E2||2^+_1\rangle$
  - ➢ χ^2 surface to look for best solution
  - ➢ Example: ^{200}Po on ^{104}Pd



^{200}Po on ^{104}Pd



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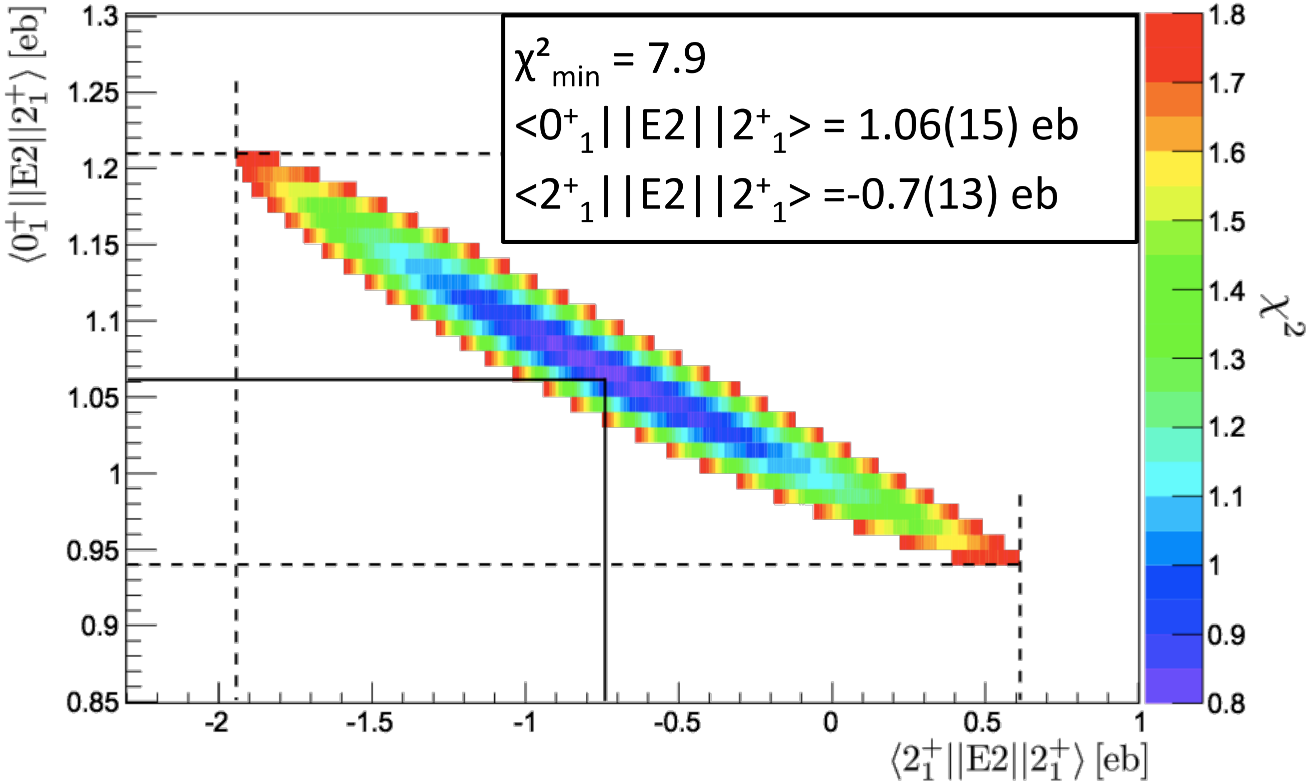
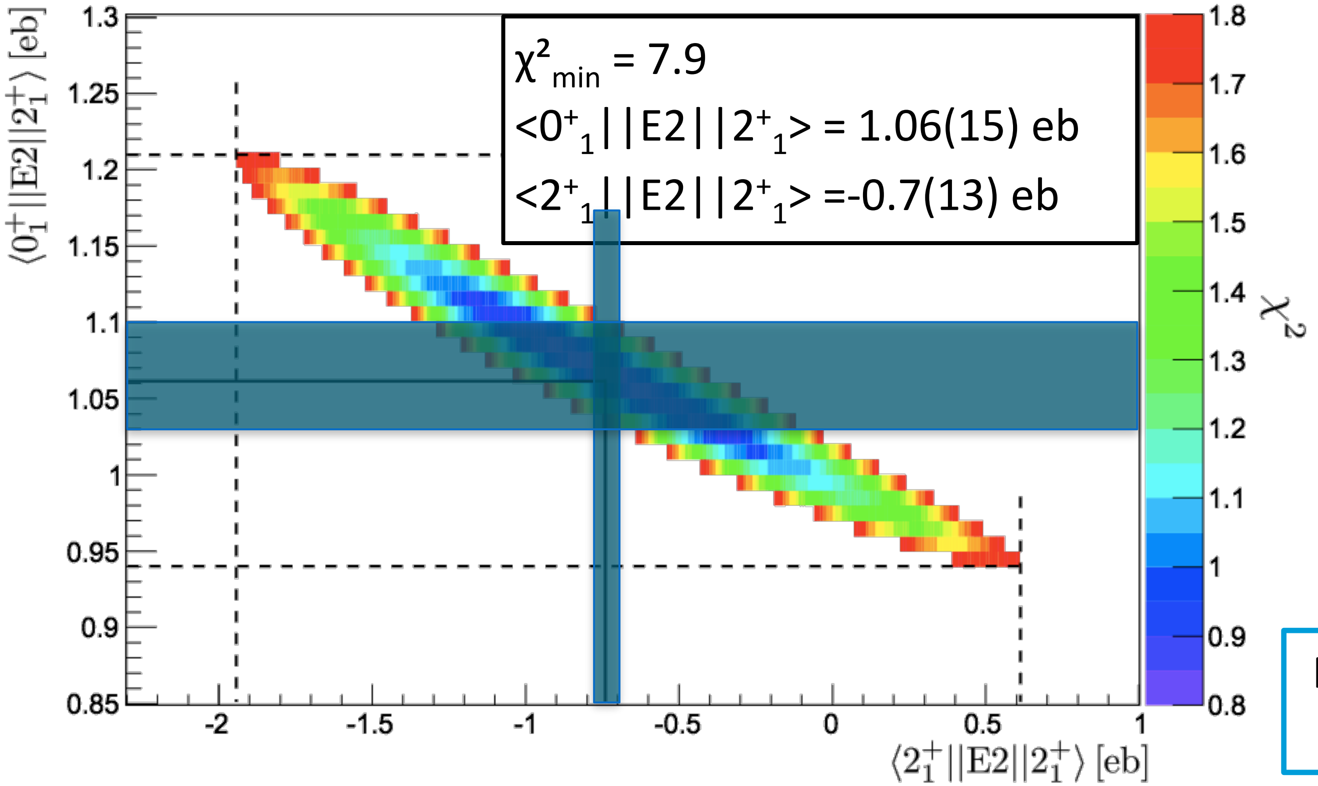


Isotope	$E_\gamma(2_1^+ \rightarrow 0_1^+)$ [keV]	$\langle 0_1^+\|\|E2\|\|2_1^+\rangle$ [eb]	$\langle 2_1^+\|\|E2\|\|2_1^+\rangle$ [eb]
^{202}Po	677.2(2)	$1.06(^{15}_{13})$	$-0.7(^{13}_{12})$
^{200}Po	665.9(1)	1.03(3)	0.1(2)

^{200,202}Po – Easy!



Factor ~3-5 on B(E2) error

Isotope	$E_\gamma(2_1^+ \rightarrow 0_1^+)$ [keV]	$\langle 0_1^+\|\|E2\|\|2_1^+\rangle$ [eb]	$\langle 2_1^+\|\|E2\|\|2_1^+\rangle$ [eb]
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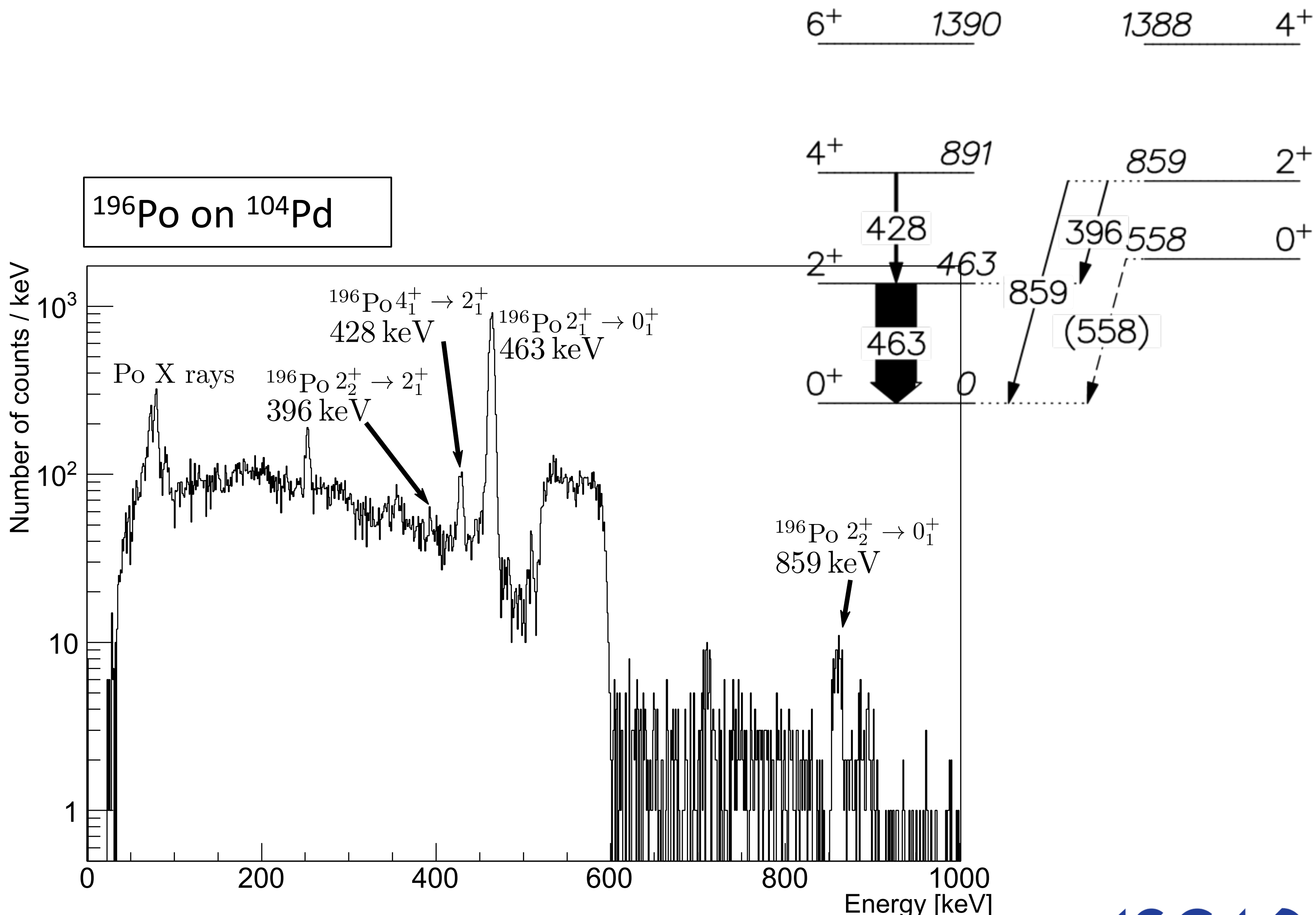
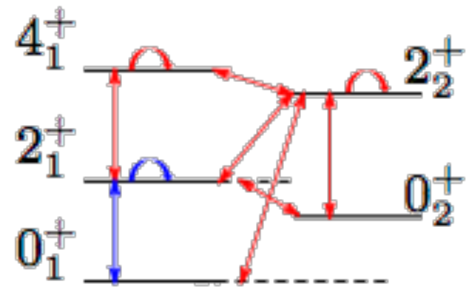
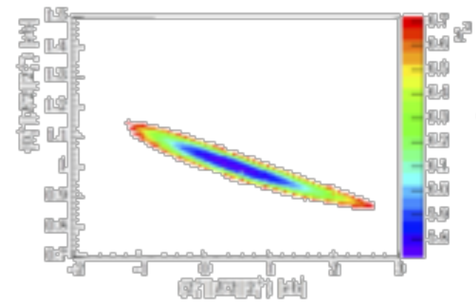
^{196}Po – Not so easy!



GOSIA-GOSIA2 method

GOSIA2; first approximation





$\langle 0_1^+ \| E2 \| 2_1^+ \rangle$

M. Zielińska, et al, *EPJA* **52**, 99 (2016).

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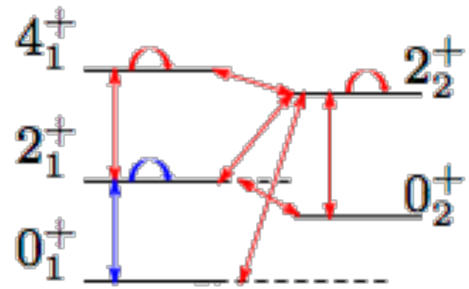
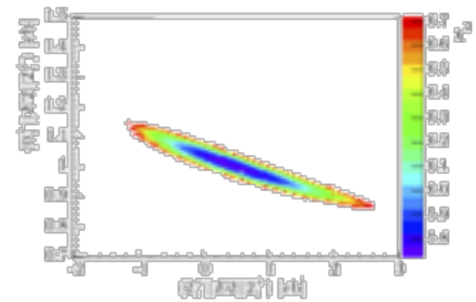


standard GOSIA; target $\xrightarrow{C_{ij}}$



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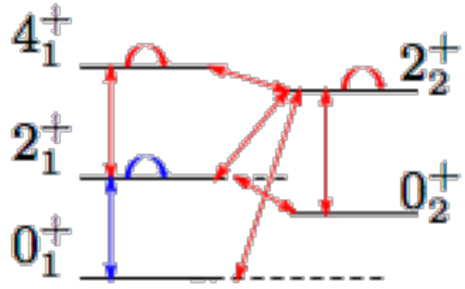
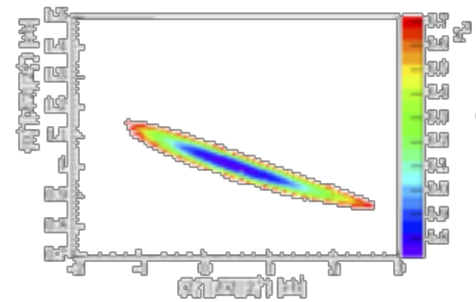


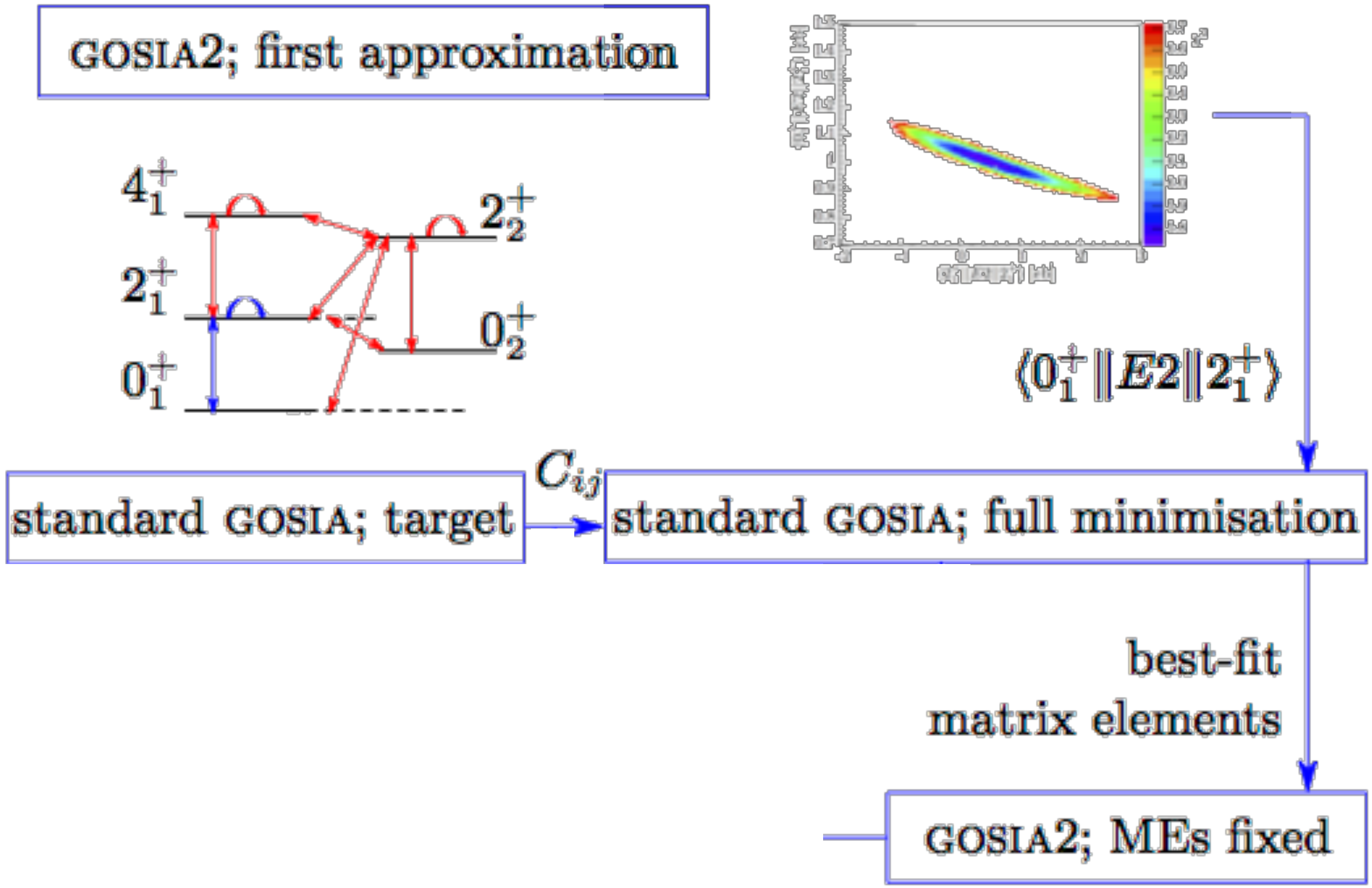


$\langle 0_1^+ \| E2 \| 2_1^+ \rangle$

standard GOSIA; target $\xrightarrow{C_{ij}}$ standard GOSIA; full minimisation

M. Zielińska, et al, *EPJA* **52**, 99 (2016).

GOSIA-GOSIA2 method



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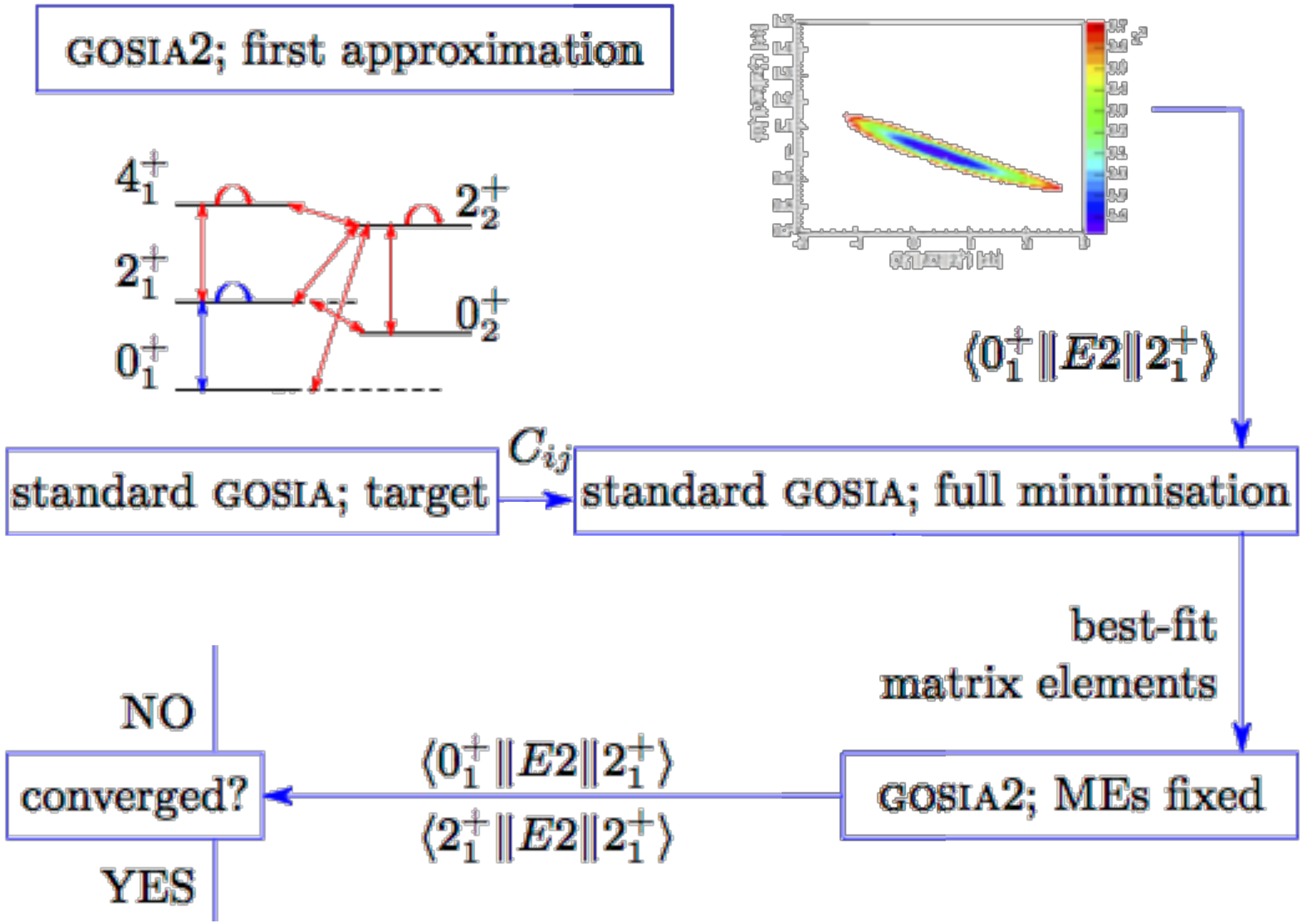

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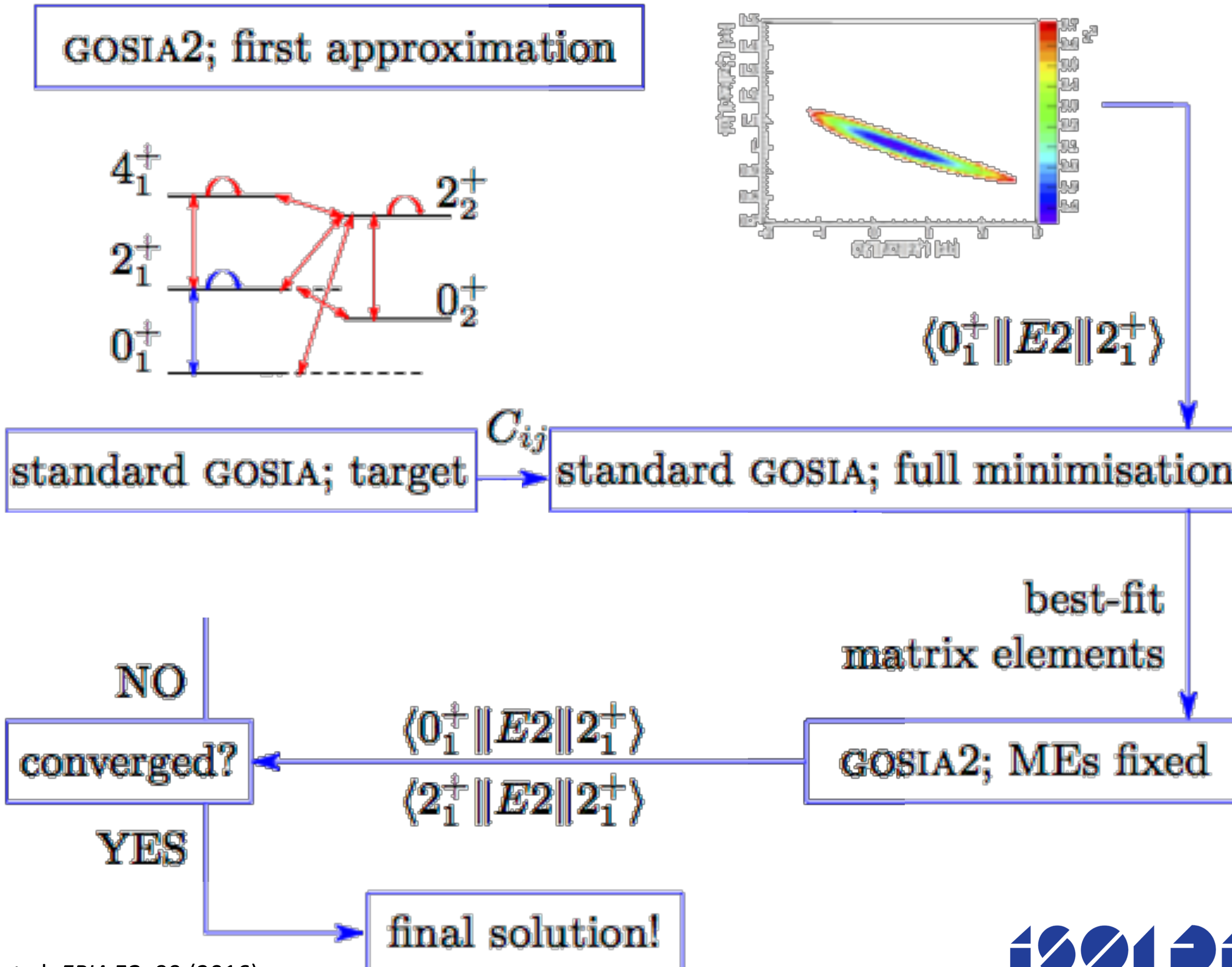

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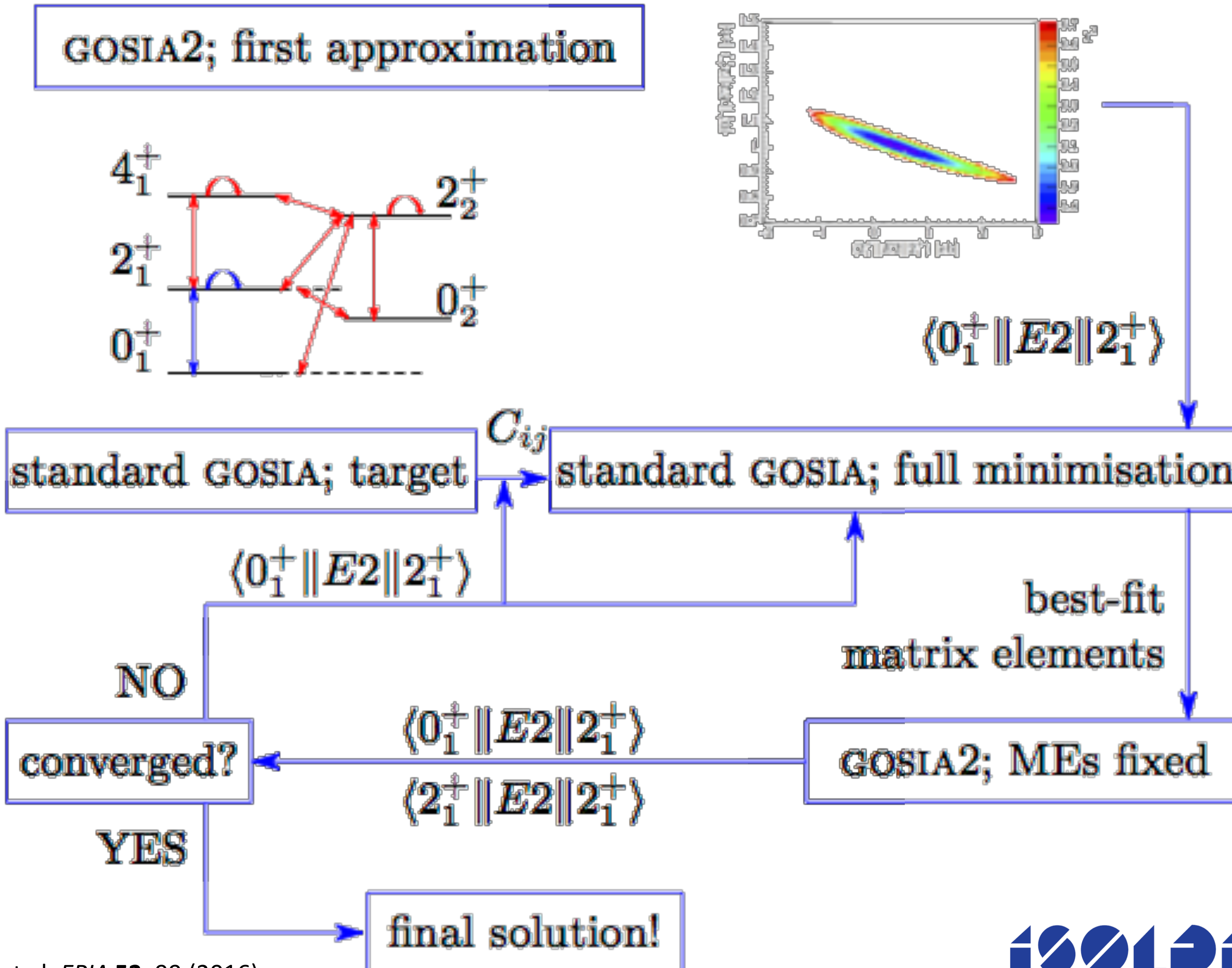
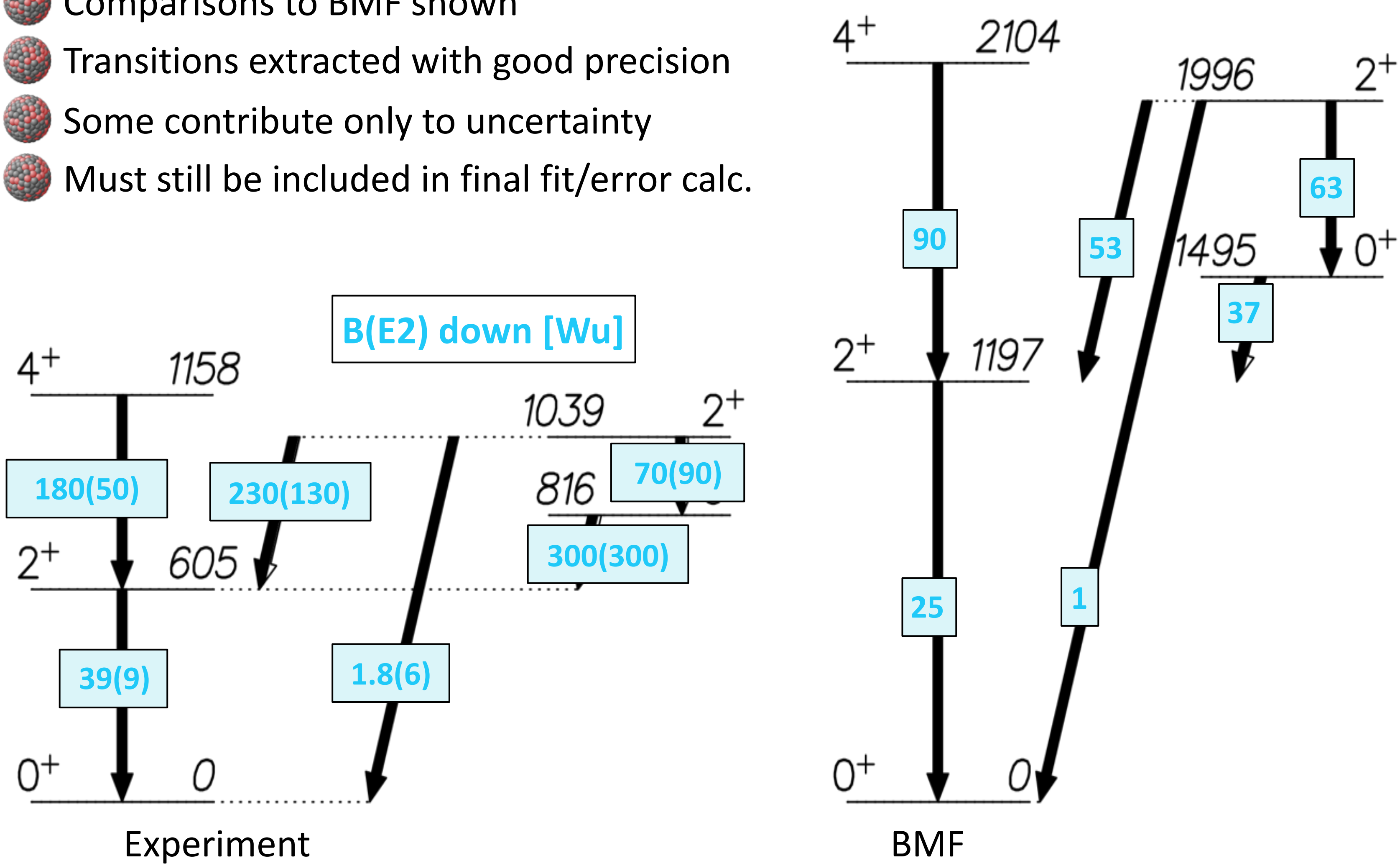


M. Zielińska, et al, *EPJA* **52**, 99 (2016).

^{198}Po – Precision & consistency

- Comparisons to BMF shown
- Transitions extracted with good precision
- Some contribute only to uncertainty
- Must still be included in final fit/error calc.



*J.M. Yao, M. Bender, P.-H. Heenen PRC **87**, 034322 (2013)*

GOSIA-GOSIA2 method

- Key things to remember:

  - Yields of normalisation transitions to include any additional uncertainties
  - Relative efficiency uncertainty included for each transition

  - Fix the relative normalisation (C_{ij}) of each EXPT using GOSIA2 values
  - Extract these from a GOSIA fit of the target data → perfect fit?
  - GOSIA normalisation should be adjusted to reproduce GOSIA2 result

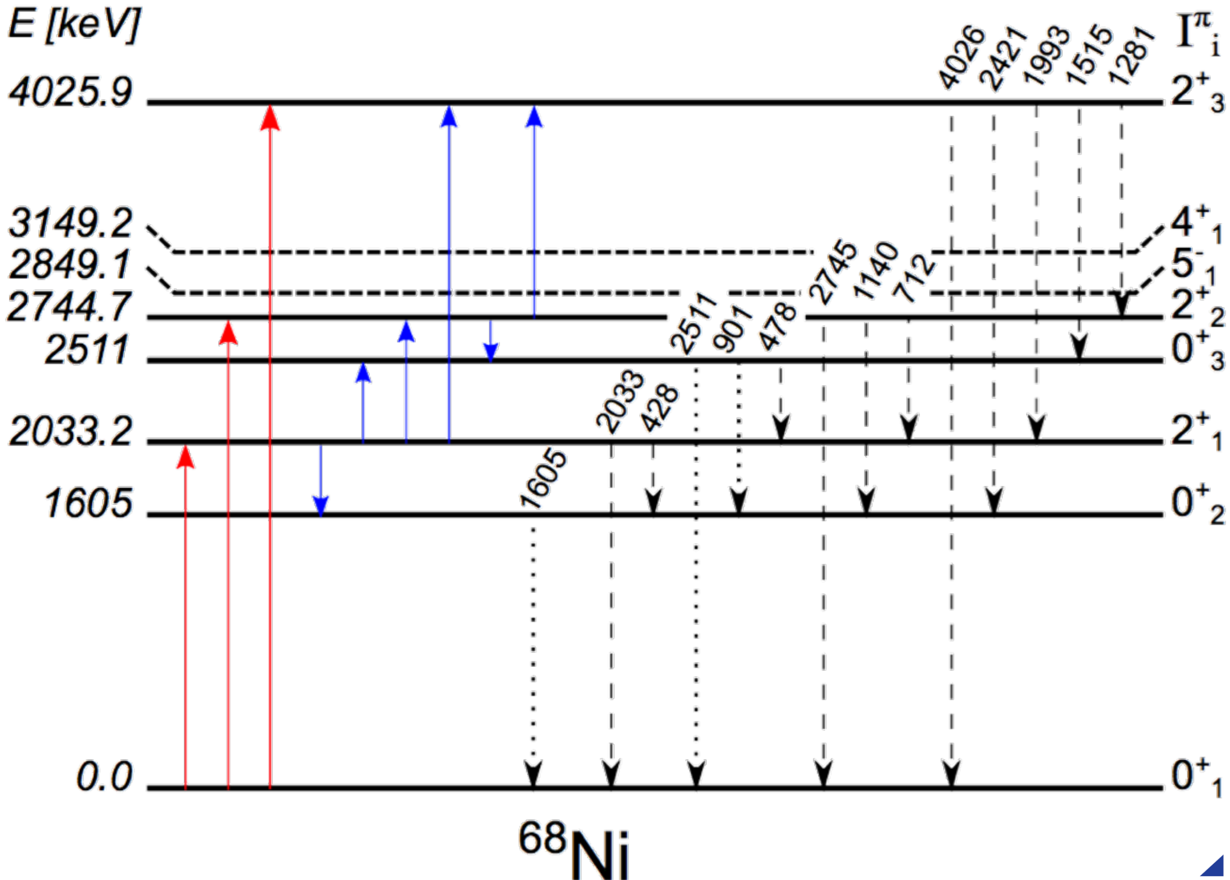
  - "Full" angular range can be included in GOSIA part of fit (norm. free)
  - Data from different targets can be included in GOSIA part of fit (norm. free)
  - Q_s sensitivity comes from fixed C_{ij} and B(E2) data, with GOSIA2 error bar!

  - Re-run full χ^2 surface for all ME changes
  - Give B(E2) from GOSIA2 as additional data point in GOSIA fit
  - No need to fiddle error bars as long as B(E2) is normalisation transition

ISOLDE

^{68}Ni - correlations

- Correlations to other states are important too… 0^+_2 state in ^{68}Ni.
- B(E2; 2^+_1 → 0^+_1) is **small**; but B(E2; 2^+_1 → 0^+_2) is **big**… *Second* 2nd order.



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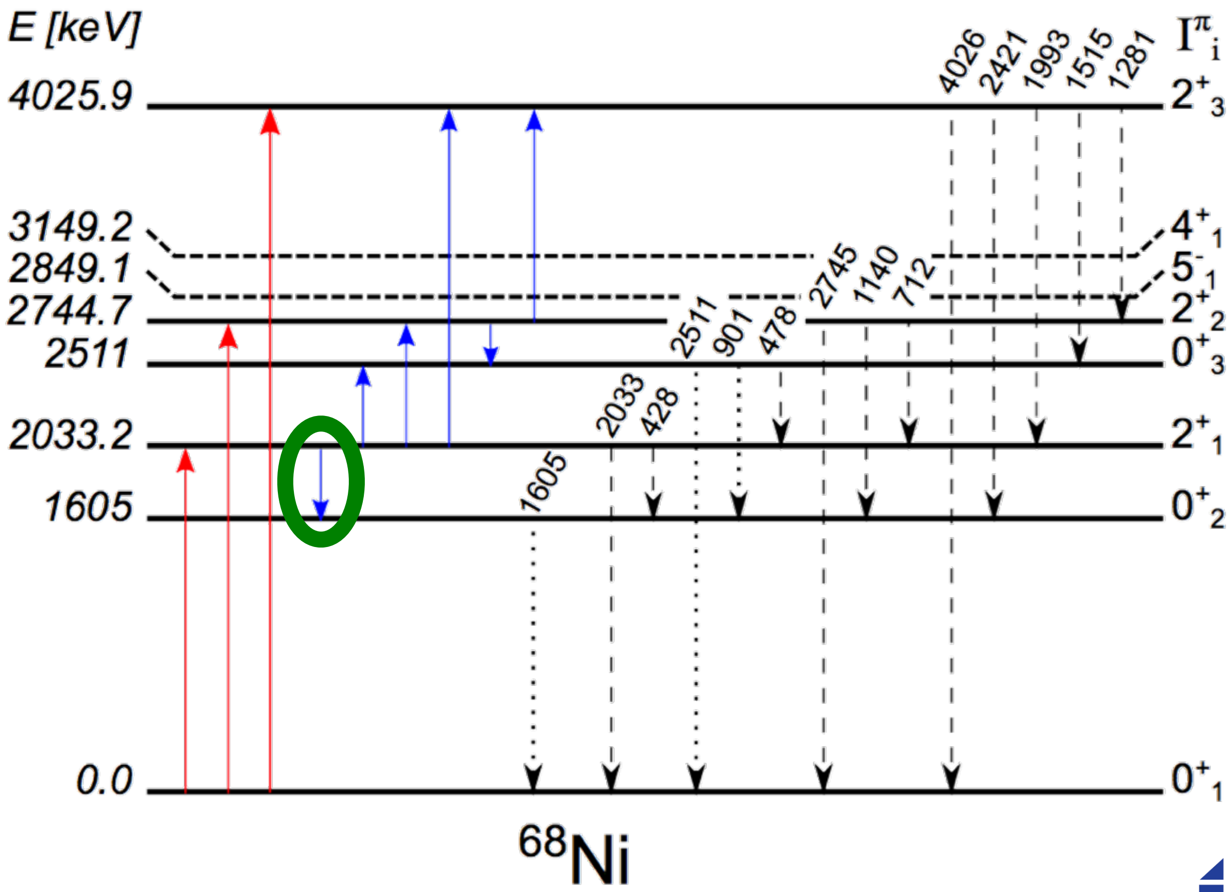
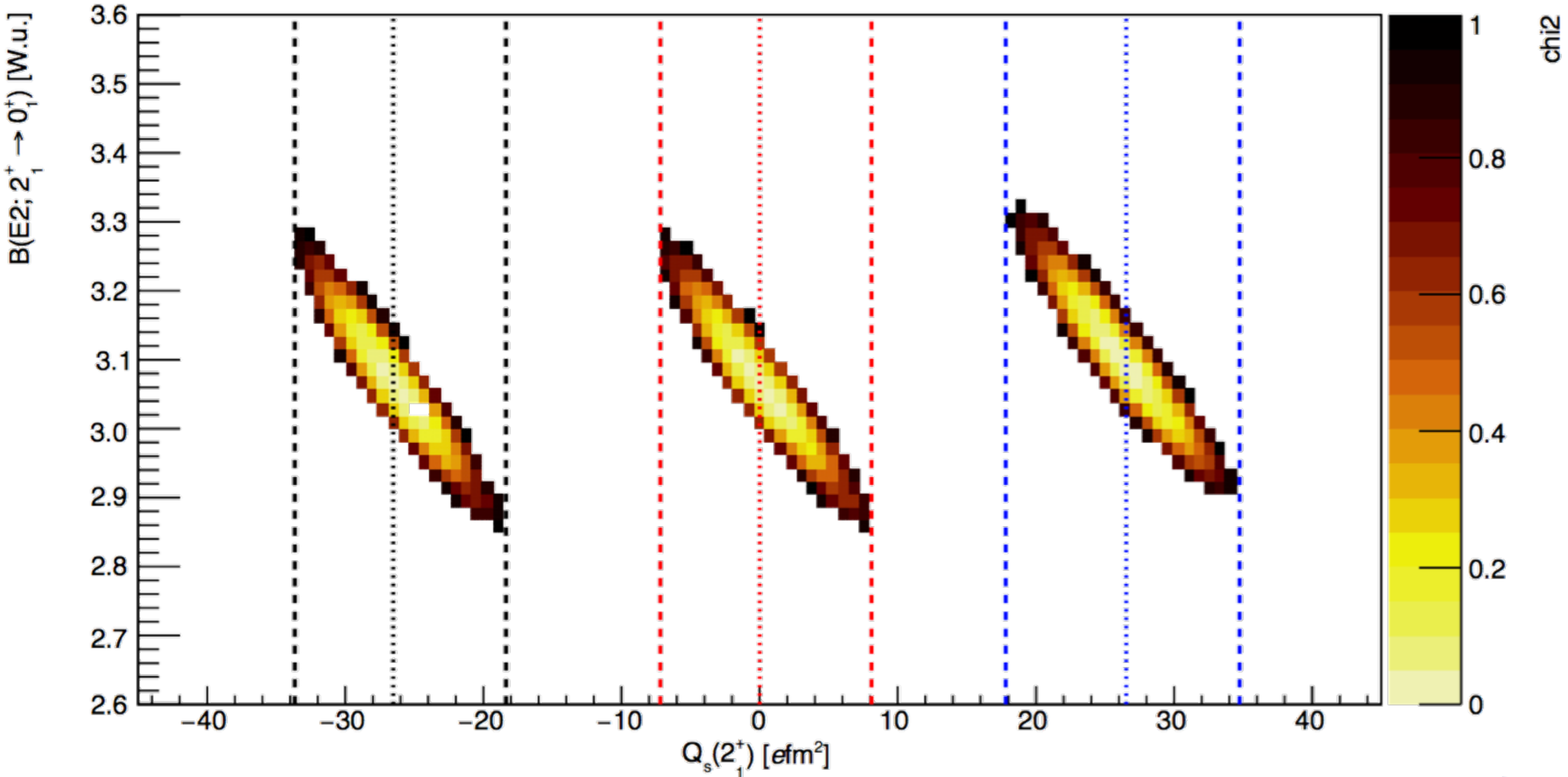


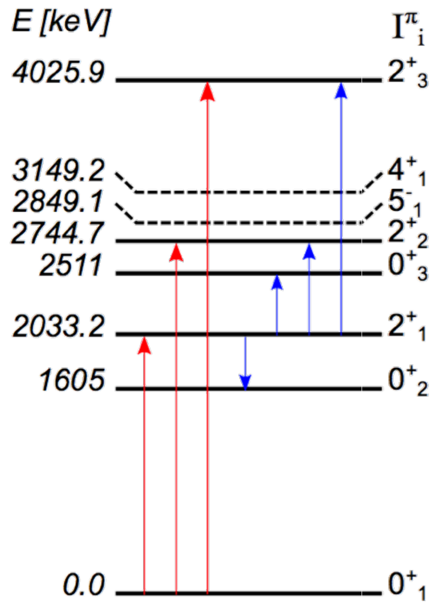
^{68}Ni – Normal 2D plot

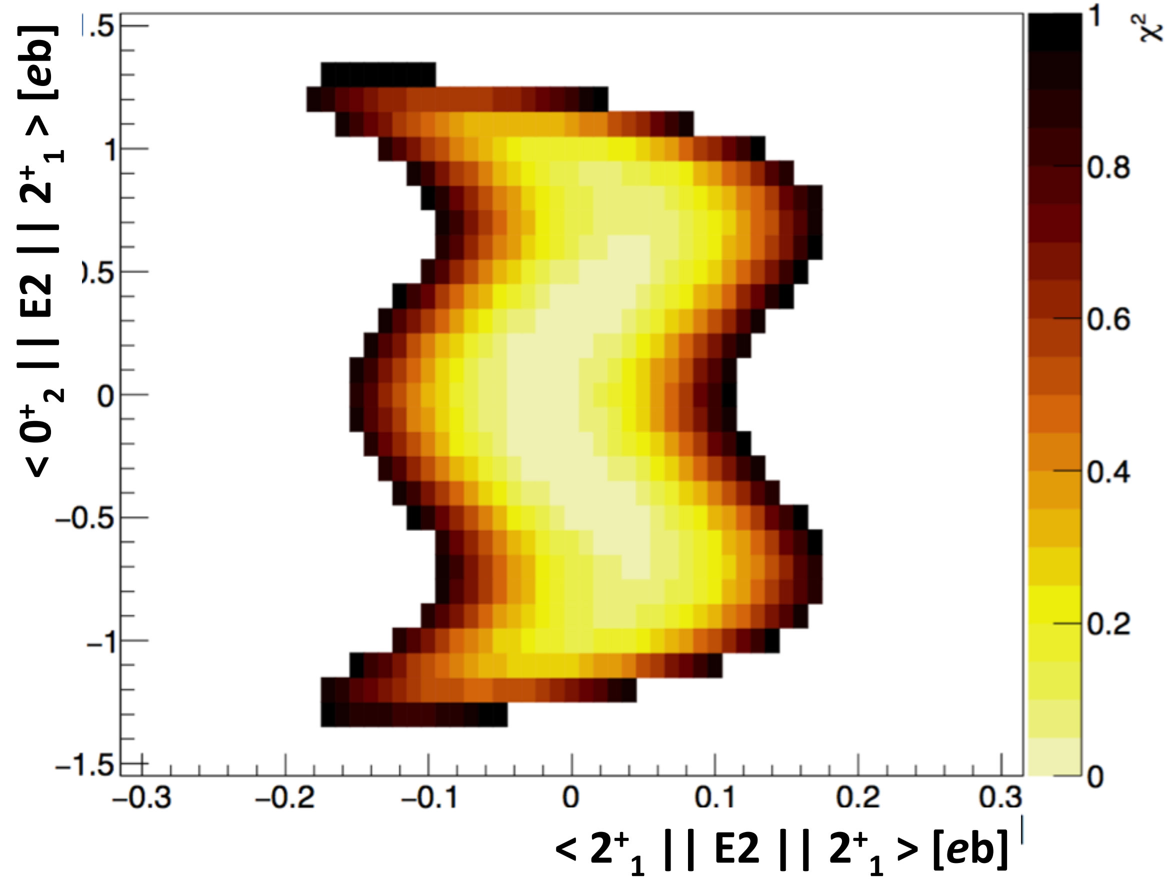
- A normal simulation of B(E2; $2^+_1 \rightarrow 0^+_1$) vs. $Q_s(2^+)$
- Three different values for $Q_s(2^+)$ are assumed.
- What about the third dimension? Look at B(E2; $2^+_1 \rightarrow 0^+_2$)



ISOLDE

^{68}Ni – The 3rd dimension

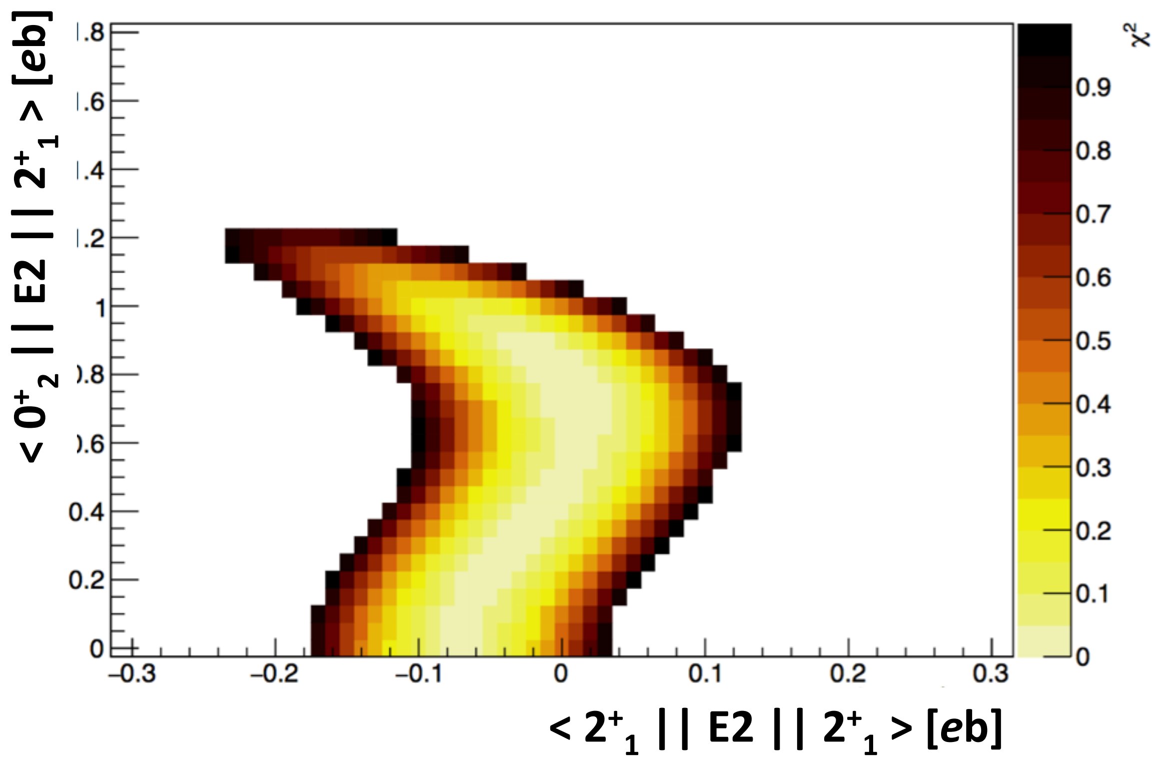




- $\tau(2^+_1)$ known!
- Project out…
- Symmetric – no sensitivity to sign
- Non-linear – but constrained

^{68}Ni – The 3rd dimension

- Starting value of $<0^+_2||E2||2^+_1>$ is important to quanitify correlation.
- Range of values to be investigated with χ^2 map for each.
- Trend is the same, but exact limits can be different.



Summary

- GOSIA2 described for use with RIBs and normalisation to target excitation.
  - Full inclusion of all uncertainties is important
  - $\chi 2$ surface scan of 2D parameter space for correlated errors

- GOSIA-GOSIA2 method used when multiple levels are excited.
  - Combines target normalisation of GOSIA2 method with full GOSIA calculation.

- Method can be used in simulations to investigate correlations
  - Any two parameters can be tested and a correlation map generated
  - `chisqsurface` code available to do the hard work!
  - Can (*should*) be updated to perform GOSIA investigations as well as GOSIA2

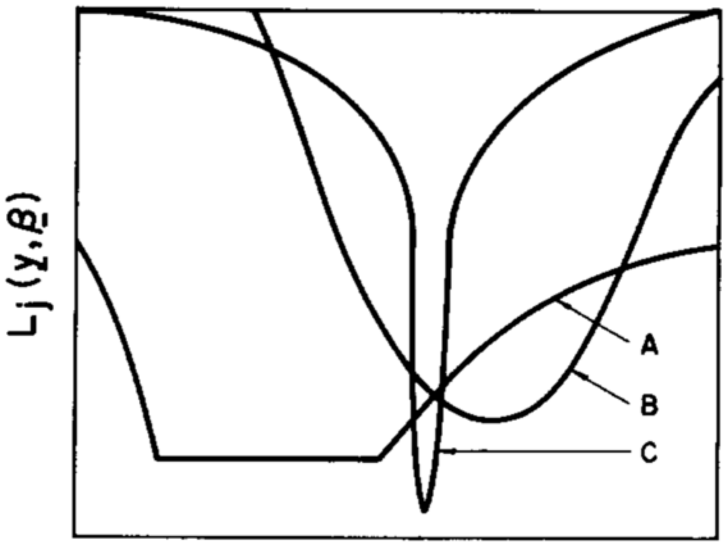
Thank you…

FIN

FIN

$\chi^2 + 1$ assumptions

- Surface *must* be parabolic about the minimum!



$$\chi^2_{\text{limit}} = \frac{\chi^2_{10\%}}{\chi^2_{50\%}} \times \chi^2_{\text{min}}$$

$$\chi^2_{\text{limit}} = \chi^2_{\text{min}} + 1$$

$$\chi^2_{\text{limit}} = \chi^2_{\text{min}} \left(1 + \frac{1}{n-p} F_\alpha(1, n-p)\right)$$

$$\chi^2_{\text{limit}} = \chi^2_{\text{min}} \left(1 + \frac{p}{n-p} F_\alpha(n, n-p)\right)$$

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ANALYTIC AND GRAPHICAL METHODS FOR ASSIGNING ERRORS TO PARAMETERS IN NON-LINEAR LEAST SQUARES FITTING†

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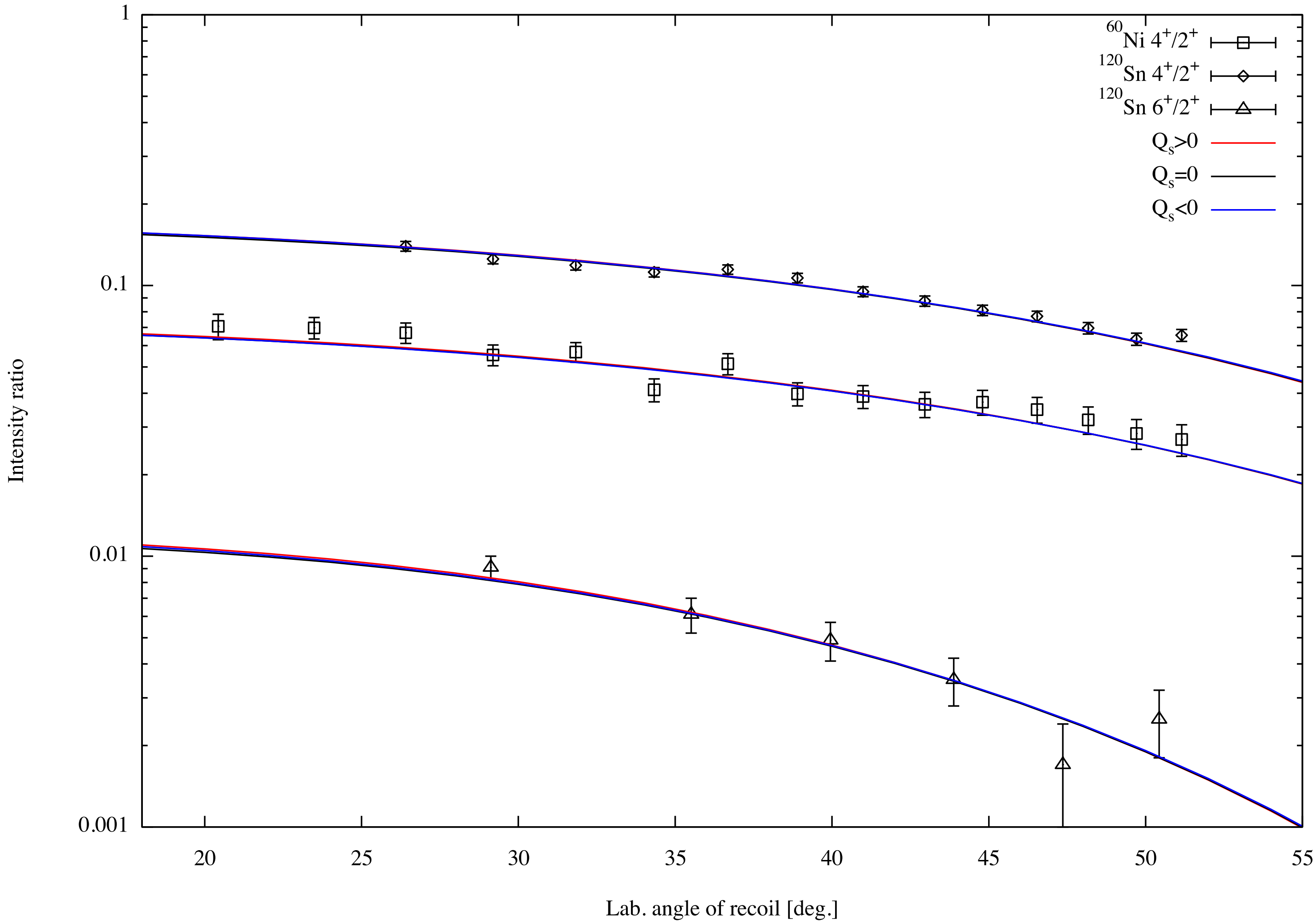
TABLE 1

Percentage of intervals including the "true" value of the parameter "b" in 300 000 simulated experiments fitting $y = a + bx$ with 5 data points and using the correct variances on the input data for case 1 and incorrect variances for case 2.

Method	Text reference	Predicted % for case 1	% of intervals including "b" Case 1[d]	Case 2[e]
Internal				
Analytic	12, 15	68.3	68.2	99.9
Graphical[f]	19, §5	68.3	68.2	99.9
External				
Analytic	13, 14, 17	68.3	68.5	68.5
Graphical[g]	21, §5	68.3	68.5	68.5
Cline and Lesser				
Graphical	([a])	90	95.4	95.4
Analytic	([b])	68.3	61.0	61.0
$\chi^2_{10\%}/\chi^2_{50\%}$	([c])	68.3	88.7	88.7
Ellipsoid[h]	22	90.0	90.0	90.0

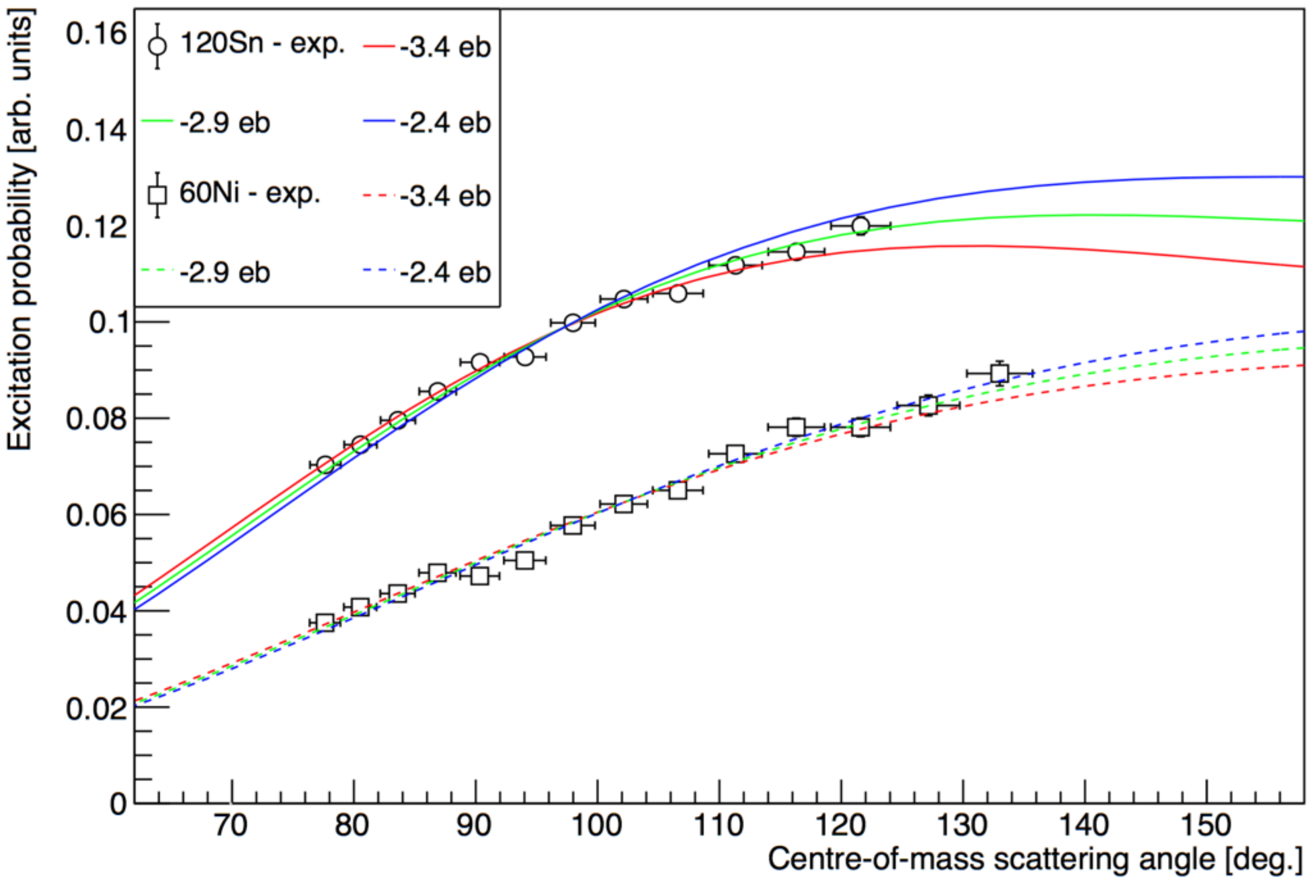
^{220}Rn – No target excitation

- Yes, we do have the 2^+_1 lifetime, so it's more traditional



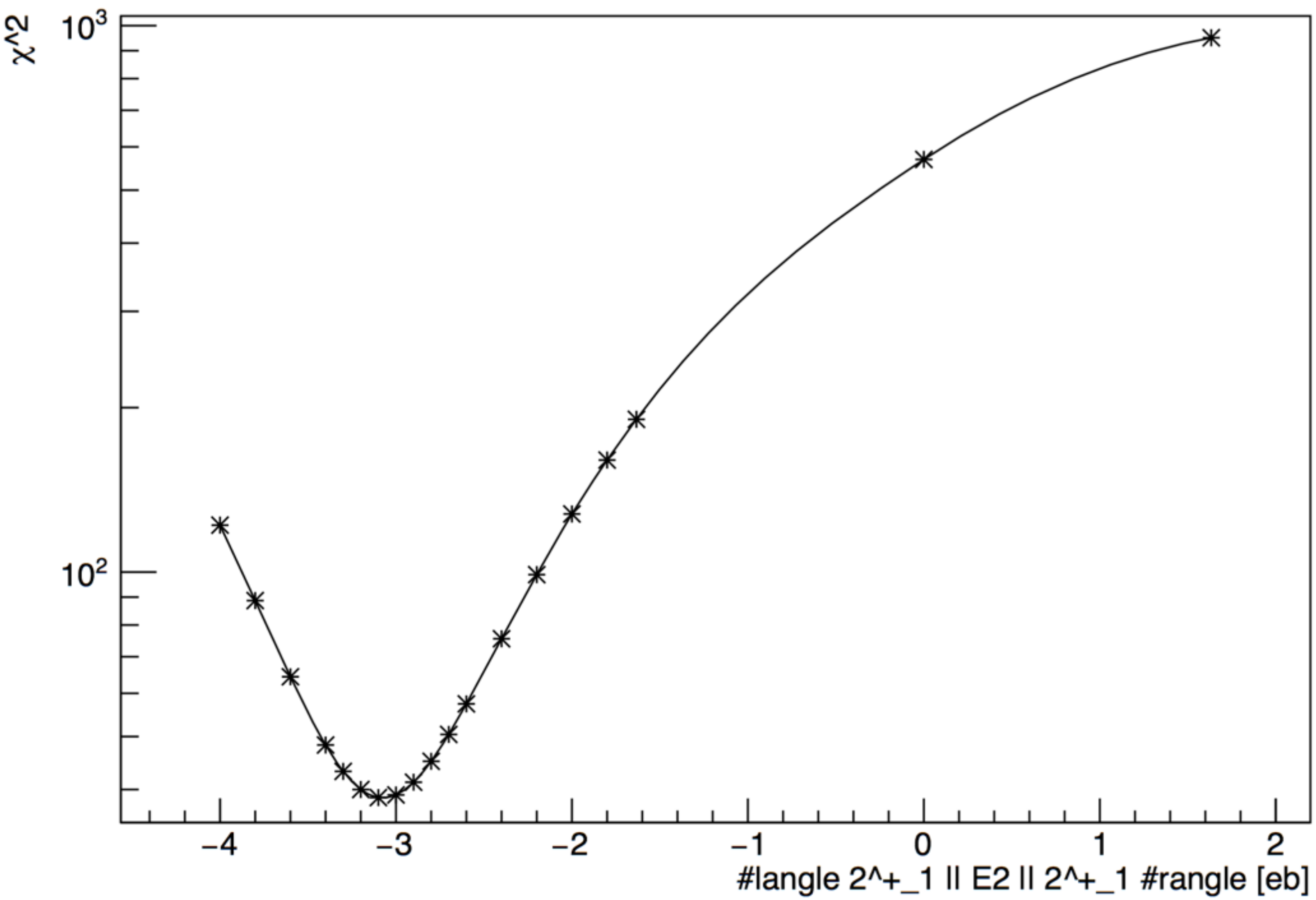
^{220}Rn – Rutherford

- Normalising the data to Rutherford XS using Gosia manual formulism
- Changes in efficiency etc corrected using *relative* particle singles intensity
- Gradient is parameter of interest… still not absolute values



^{220}Rn – $Q_s(2^+_1)$ sensitivity

- $\chi 2$ scan of possible $<2^+_1||E2||2^+_1>$ reveals true sensitivity
- Rigid-rotor value is 1.63 *e*b and best fit is almost double… Why?



^{220}Rn – Rutherford

- Not as easy as it seems: Very sensitive to angle determination (CD distance!)

In this case, a 2 mm shift in CD-target distance = ~1 eb change in <2^+_1||E2||2^+_1>